UNSTEADY-STATE PERFORMANCE OF WATER-DRIVE GAS RESERVOIRS

A Dissertation

By

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Unsteady-State Performance of Water-Drive
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ABSTRACT

Although it has long been realized that gas recovery from a water-drive gas reservoir may be poor because of high residual gas saturation left in the water-invaded portion of the reservoir, it appears that only limited information on the subject has been available until recently.

This study was made to: (1) develop methods for forecasting the unsteady-state performance of water-drive gas reservoirs, (2) investigate the quantitative effect of various reservoir parameters which may influence and control the recovery of gas from such reservoirs, and (3) look into the possibility of obtaining a correlation between the residual gas saturation and other reservoir rock parameters. The study was performed in the main with a high-speed digital computer so that performance of water-drive gas reservoirs could be evaluated for a large variety of conditions.

Results indicate that gas recovery from such reservoirs may be very low in some cases; perhaps as low as 45 per cent of the initial gas in place. Gas recovery under water drive appears to depend in a significant way upon: (1) field production rate and manner of production, (2) aquifer properties, (3) volumetric displacement efficiency, and (4) residual gas saturation behind the water invading the gas reservoir.

The manner of estimating water influx in a water-drive gas reservoir can vary considerably. Examples are: the steady-state method, the Hurst modified steady-state method, and various unsteady-state methods such as those of van Everdingen-Hurst, Hurst and Carter-Tracy. In this study, the Carter-Tracy water-influx expression has been used mainly, but results have also been obtained by using a rigorous superposition method. The comparison was found to be excellent.

Multiple regression analysis techniques were used to study the relationships between imbibition residual gas saturation and parameters such as porosity, permeability, and initial gas saturation. Data points used in this study included published and unpublished determinations of imbibition residual gas saturation. Results of the several attempts to correlate the data indicated that it was not possible to develop a general correlation of high accuracy. However, several useful regression equations for the residual gas saturation were developed.

In regard to the performance of water-drive gas reservoirs, it appears that in certain cases gas recovery can be increased significantly by controlling the field production rate and manner of production. For this reason, an early investigation of water influx should be made in particular gas reservoirs to permit adequate planning to optimize the gas reserves.

INTRODUCTION

In recent years, the economic importance of natural gas production has increased at a tremendous rate. This has been evidenced by intensive exploration efforts aimed at gas production, and exploitation of deep, as well as low-permeability, gas reservoirs. The increased importance of natural gas has also dictated the storage of natural gas in underground reservoirs to solve inventory and marketing problems. Since many of the gas storage reservoirs are in aquifers, and a large number of gas reservoirs are under water drive, a considerable effort is being expended in applying engineering principles to the study of water-drive gas reservoir performance in general, and storage-field behavior in particular. Unfortunately, water influx has often forced abandonment of gas wells at extraordinarily high pressures. For this reason, a study of the performance of water-drive gas reservoirs is needed to permit adequate planning of gas reservoir production.

Many engineering methods have been developed whereby the size and performance of oil reservoirs may be analyzed mathematically. The use of the material balance equation to estimate the volume of hydrocarbons originally present in a reservoir, with or without water drive, has been discussed by many authors. Schilthuis[1] developed the present day energy equation for hydrocarbons. The work of Hurst,[2] and van Everdingen and Hurst[3] permitted consideration of unsteady water influx into the oil reservoir. Clearly, methods developed for oil reservoirs should be susceptible to modification for gas reservoirs.

When reservoir performance data are available, it may be possible to match a mathematical model to field performance data to determine the initial gas in place, and the water influx parameters: the dimensionless time constant, the water influx constant, and sometimes aquifer size and approximate geometry. Least-mean-square fitting techniques may be applied to a rigorous gas material balance equation to determine the above mentioned parameters. However, it should be mentioned that recently, Chierici, _et al_.,[4] reported that gas reserves can _not_ be unequivocally determined from the past performance of partial water-drive gas reservoirs. Many plausible reservoir models may match the same field data equally well.

So far, no study has been made concerning the variations in reserves which may be obtained by using the gas reservoir performance equation in different algebraic forms. There is definite need for a study of the most

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[1]References given at end of dissertation.

suitable form of the performance equation useful for determining the initial gas in place and the water influx parameters.

Many methods are available for estimation of water influx. Examples are: the steady-state method,[1] the Hurst modified steady-state method,[2] and various unsteady-state methods such as those of van Everdingen-Hurst,[3] Hurst,[5] and Carter-Tracy.[6] Most of the available methods utilize the principle of superposition for variable rate performance calculations. Applications of these solutions have appeared sporadically[7-11] in the literature.

The experimental study of residual gas saturations under water drive by Geffen, et al.,[12] in 1952 indicated that residual gas saturations could be extremely high. A value of 35 per cent of pore volume is often used in field practice when specific information is not available. Naar and Henderson[13] concluded that the residual non-wetting phase saturation under imbibition should be about half of the initial non-wetting phase saturation. This value appears to be a good estimate if laboratory measurements are not available.

After the pioneer work of Geffen, et al., several groups carried out experiments and reported values of residual gas saturation.[14-19] From their studies, it is clear that a considerable portion of the initial gas in place might be trapped in a water-drive gas reservoir at high pressure. A full water-drive would result in loss of residual gas trapped at initial reservoir pressure. Consideration of transient aquifer behavior leads to the conclusion that high rate production of water-drive gas reservoirs could result in improved gas recovery by the reduction of the abandonment pressure. However, there appears to be little quantitative information regarding the effect various reservoir parameters might have on the recovery of gas from water-drive gas reservoirs.

One of the few advantages of water-drive gas production appears to be improved deliverability through support of the reservoir pressure. There may also be an advantage in higher condensate recovery caused by pressure maintenance for gas condensate water-drive reservoirs.

Because of the difficulty and time involved in measuring residual gas saturation under water imbibition, it is a common field practice to assume a value of 25 or 35 per cent of pore volume for residual gas saturation. Since the true value may be anything from 15 to 50 per cent of pore space, this may lead to gross errors in estimates of reserves for a water-drive gas reservoir.

Chierici, *et al.*,[17] later Katz, *et al.*,[18] tried to establish a relationship such that residual gas saturation could be predicted on the basis of petrophysical characteristics of reservoir rocks. Neither was successful.

The purpose of the present study was: (1) to develop methods for forecasting the unsteady-state performance of water-drive gas reservoirs; (2) to investigate the quantitative effect of various reservoir parameters which might influence the recovery of gas; (3) to investigate the possibility of correlation of imbibition residual gas saturations and other reservoir rock parameters.

The Carter-Tracy[6] water-influx expression has been used in this study because of its advantages in hand calculation. However, calculations have been performed in the main with a high-speed digital computer to evaluate the effect of water influx in water-drive gas reservoirs under a large variety of conditions.

REVIEW OF LITERATURE

Several methods have been presented in the past which can be applied to estimate water influx in water-drive gas reservoirs.

Schilthuis[1] was the first to develop useful expressions for calculating water influx in a hydrocarbon reservoir. His steady-state expression is:

$$W_e = C_s \int_0^t (p_i - p)dt \quad * \tag{1}$$

where C_s is the water influx constant in barrels per day per pound per square inch, and $(p_i - p)$ is the boundary pressure drop in pounds per square inch.

Analysis of water expansion into a hydrocarbon reservoir indicates that water influx should often be an unsteady-state process. Hence the Hurst modified steady-state equation[2] should give better results. The equation is:

$$W_e = C_h \int_0^t \frac{(p_i - p)}{\log at} dt \tag{2}$$

where C_h is the water influx constant in barrels per day per pound per square inch, and <u>a</u> is a time conversion constant which depends upon the units of time, t.

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*Symbols are listed beginning on page 68.

Various rigorous unsteady-state methods[3,5,6] are now available which can be used to predict the performance of water-drive gas reservoirs. The van Everdingen-Hurst[3] unsteady-state method is a satisfactory method for calculating water influx, but it requires the use of superposition to permit variable rate performance calculations. Hence it is difficult to use without a digital computer. The Hurst[5] unsteady-state expression is:

$$W_e = B \int_0^{t_D} \frac{d\Delta P}{d\tau} Q_D(t_D - \tau) d\tau \qquad (3)$$

where B is the water influx constant in barrels per pound per square inch, and $Q_D(t_D)$ is the dimensionless cumulative influx as a function of dimensionless time, t_D. Hurst solved Eq. 3 using the Laplace Transformation and provided a method which has the advantage that it does not require tedious superposition calculations if the field production rate is constant. The method has limited application for gas reservoir studies.

The Carter-Tracy[6] unsteady-state expression (to be presented later in the text) offers some advantages in hand calculation which do not appear to have been generally recognized. This equation can be combined conveniently with a suitable material balance equation to predict the performance of water-drive gas reservoirs.

The application of the above cited methods of water-drive gas reservoir and aquifer gas storage-field problems has been sporadic[7-11] at best. Water-drive gas reservoir performance can be predicted by combining a gas material balance for the reservoir and a suitable water-influx equation for the aquifer. One of the problems encountered in water-drive gas reservoirs is to determine the initial gas in place and the water influx parameters from field performance data. An interesting investigation of the uncertainty in reserve estimation of water-drive gas reservoirs from past history, has appeared recently.[4] Chierici, et al., used the performance equation in the form of a straight line as reported by several other authors. However, no investigation has been made of variations in hydrocarbon reserve figures obtained by using the same performance equation in different algebraic forms.

Another problem encountered in water-drive gas reservoir performance is that a high residual gas saturation is left in the water invaded portion of the gas reservoir under imbibition. Because of the study by Geffen, et al.,[12] reported in 1952, it has long been known that gas recovery should be poor for water-drive gas reservoirs. This results because of the high residual gas saturation under water imbibition, coupled with the possibility of leaving residual gas at high pressure at abandonment (although Cole[20] makes the peculiar implication that gas recovery under water drive should be higher than under volumetric expansion). Because of the possibility of trapping residual gas at high pressure, it has been generally recognized that high field production rates might increase gas recovery by reducing pressure in residual gas at abandonment. Most gas producers will "pull hard" on wells in fields thought to be under water drive. However, there appears to be no quantitative description of just what this means.

In regard to experimental studies of residual gas saturations, Geffen, et al.,[12] found that the values of residual gas saturations at 5000 psia and 250°F were very close to those obtained at atmospheric conditions. Their results indicated that the residual gas saturations under water drive could be 30 to 40 per cent of pore volume.

Elliott,[14] and Kruger[15] studied the effect of initial gas content on the residual gas saturation of cores after water-flooding.

Naar and Henderson[13] concluded that residual non-wetting phase saturation under imbibition should be about half the initial non-wetting phase saturation.

In 1962, Khudyakov and Velikovskii[16] carried out experiments on recovery of gas by water displacement.

They obtained more or less the same results as obtained by earlier investigators.

The experimental study of residual gas saturations under water drive by Chierici, _et al._,[17] in 1963 on unconsolidated sands, sandstones and bioclastic limestones confirmed that residual gas saturation could reach 30 to 40 per cent of pore volume; these values are in the same range as residual oil saturations for oil reservoirs produced by water drive.

In 1966, Katz, _et al._,[18] and Crowell, _et al._,[19] published experimental determinations of residual gas saturations. Their conclusions were similar to those of the previously-cited investigators.

The measurement of residual gas saturation under water imbibition is difficult and time consuming. Chierici, _et al._,[17] and Katz _et al._[18] tried to find a relationship between residual gas saturation values and petrophysical characteristics of rocks that would allow the calculation of residual gas saturation, but were not successful in their attempt.

PERFORMANCE EQUATIONS

The unsteady-state performance of water-drive gas reservoirs has been derived in this study in a manner analogous to oil reservoir calculations: a material balance is written for the reservoir, and a water influx equation is written for the aquifer. Simultaneous solution provides the cumulative water influx and the reservoir pressure.

A general form of material balance for a water-drive gas reservoir is:

$$G(B_{gn} - B_{gi}) + W_{en} - B_w W_{pn} = G_{pn} B_{gn} \tag{4}$$

It should be noted that gas formation volume factors for Eq. 4 are expressed in reservoir barrels per standard cubic foot, while water formation volume factors are expressed in reservoir barrels per surface barrel. Often, B_w is assumed to be unity. The gas formation volume factor may be written:

$$B_{gn} = \frac{T p_{sc} Z_n}{5.615\ T_{sc} p_n} \tag{5}$$

The Carter-Tracy[6] approximate water influx equation is:

$$W_{en} = W_{e(n-1)} + (t_{Dn} - t_{D(n-1)})\left\{\frac{B\Delta p_n - W_{e(n-1)}p'_D(t_{Dn})}{p_D(t_{Dn}) - t_{D(n-1)}p'_D(t_{Dn})}\right\} , \quad (6)$$

where

$$t_{Dn} = \frac{2.31\ k_w t_n}{\phi\mu_w c_t r_w^2} , \quad (7)$$

$$B = 1.1191\ \phi c_t h r_w^2 F , \quad (8)$$

$$p'_D(t_D) = \frac{d}{dt_D}[p_D(t_D)] \quad (9)$$

The first derivative of the van Everdingen-Hurst[3] dimensionless pressure, $p_D(t_D)$, is not a common function. It is presented in Fig. 1 for an aquifer of infinite extent. Edwardson, et al.,[21] have published curve-fit equations to calculate $p'_D(t_D)$ for the infinite radial aquifer. These equations are of high precision, and are particularly useful for digital computer solution of transient flow problems. If the aquifer is of finite extent, the van Everdingen-Hurst solutions can be differentiated for times when the boundary effect is felt. The derivative will approach a constant value in this case. Curve fit equations for dimensionless pressures have also been published by Bruns, et al.,[11] for finite radial aquifers.

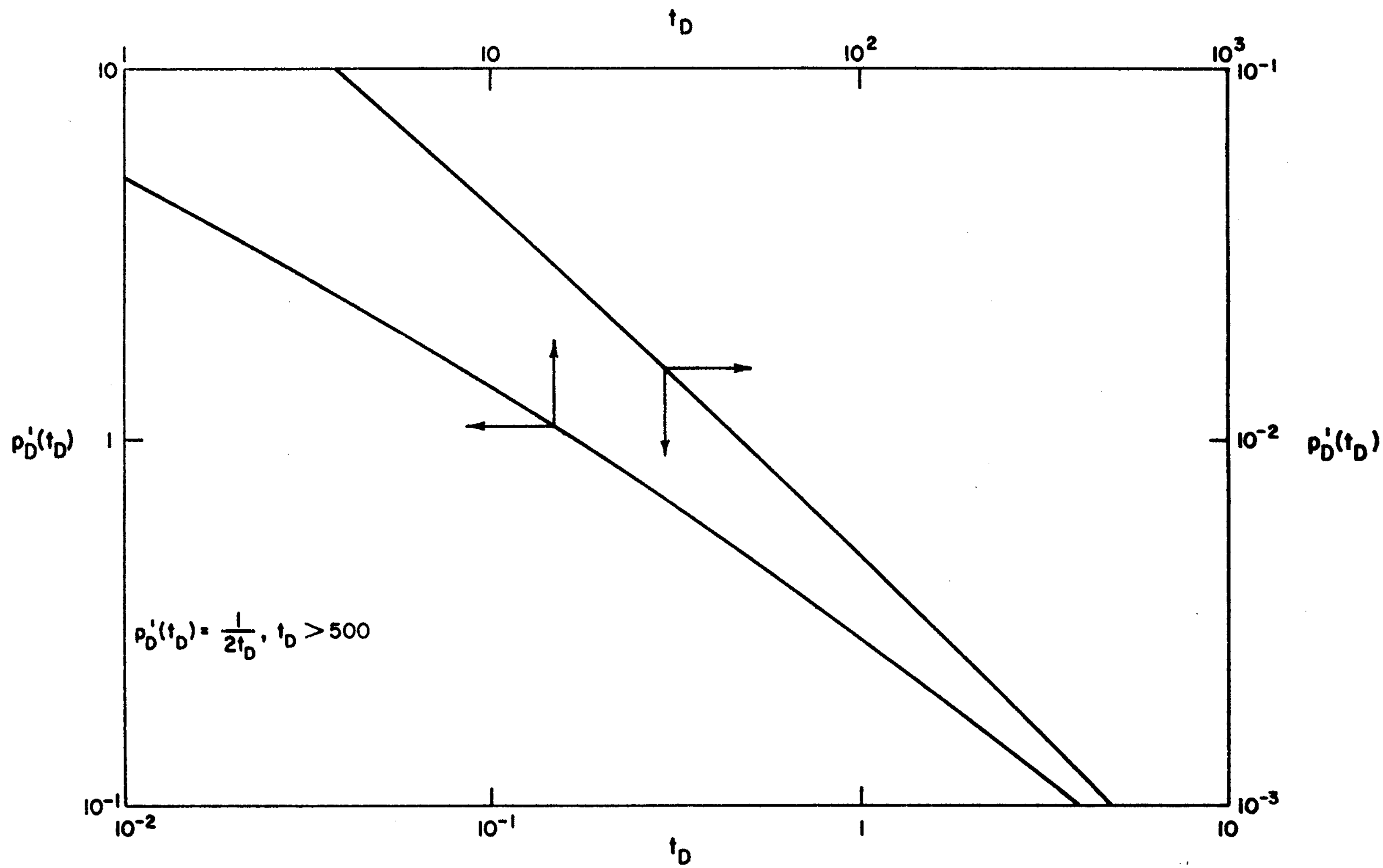

FIG. 1 – $p_D'(t_D)$ VS. t_D FOR AN INFINITE RADIAL AQUIFER

The usual transient flow superposition equation for water influx, common in reservoir engineering is:

$$W_{en} = B \, \Sigma \, \Delta p_n Q_D(t_{Dn}) \quad . \tag{10}$$

See van Everdingen and Hurst.[3] Before Eq. 10 can be used, it is necessary to specify the nature of the dimensionless cumulative influx term, $Q_D(t_D)$. This quantity has recently been generalized and called the Aquifer Response Function. In general, it depends upon the flow geometry, rock and fluid properties, and time. Although it is possible to determine the aquifer response function for any specific field case, the aquifer response functions for three potentially-useful flow geometries (hemispherical, linear and radial) are presented in the Appendix.

Eqs. 4 and 6, or Eqs. 4 and 10 can be solved simultaneously to eliminate the water influx term and provide an equation relating the pressure, p_n, to the cumulative fluid produced (commonly gas and water) at time n. Herein, mainly the Carter-Tracy influx equation, Eq. 6, has been used. The main utility of the Carter-Tracy influx equation over the usual transient flow superposition equation, Eq. 10, is that it permits explicit solutions for p_n and W_{en}. Water influx at any time is related to the water influx at the preceding time analytically.

The solution for the pressure p_n obtained from simultaneous solution of Eqs. 4 and 6 is:

$$p_n = \frac{1}{2C_nB}\{-E_n + \sqrt{E_n^2 + 4(G-G_{pn})KZ_nC_nB}\} \tag{11}$$

The derivation of Eq. 11 is presented in the Appendix. In this derivation

$$C_n = \frac{[t_{Dn} - t_{D(n-1)}]}{[p_D(t_{Dn}) - t_{D(n-1)}p_D'(t_{Dn})]} \; , \tag{12}$$

$$K = \frac{Tp_{sc}}{5.615\ T_{sc}} \; , \tag{13}$$

$$E_n = GK\frac{Z_i}{p_i} + B_wW_{pn} - W_{e(n-1)}[1 - C_np_D'(t_{Dn})] - C_nBp_i \tag{14}$$

Eq. 6 can be rearranged using the same nomenclature as Eqs. 13 through 14 to yield:

$$W_{en} = W_{e(n-1)}[1 - C_np_D'(t_{Dn})] + C_nB\Delta p_n \; , \tag{15}$$

and

$$\Delta p_n = p_i - p_n \tag{16}$$

Eq. 11 can be solved for the pressure at time t_n for gas production G_{pn}, and then water influx can be computed from Eq. 15. Solution of Eq. 11 is complicated by the fact that the gas-law deviation factor, Z_n, is a function of p_n. Thus, solving Eq. 11 requires trial and error, unless an analytical relationship between p_n and Z_n can be written. However, a digital computer method[22] is available by which Z_n can be calculated as a function of pseudo-reduced pressure, p_{pr}, and pseudo-reduced temperature, T_{pr}, with reasonable accuracy. In some cases, Z_n can be considered a linear function of p_n over increments of pressure. An expression similar to Eq. 11 can be written which does not involve Z_n. However, this usually is not particularly time saving. The trial solution of Eq. 11 is a simple trial and does not often involve more than two steps. This trial is even feasible as a hand calculation.

To set the end point or abandonment condition, a material balance equation which states that the maximum gas recovery is equal to the initial gas in place, less gas trapped as residual gas in the watered region, less gas in regions not swept by water, but unavailable to production because of breakthrough of water into all existing producing wells, is:

$$G_{pm} = G[1 - E_p\{\frac{S_{gr}}{S_g} + \frac{(1-E_p)}{E_p}\}\frac{p_m Z_i}{p_i Z_m}] \qquad (17)$$

The derivation of Eq. 17 is presented in the Appendix. The subscript "m" indicates the end point, or maximum production condition. Eq. 17 can be rearranged to:

$$\frac{p_m}{Z_m} = \frac{(p_i/Z_i)}{E_p[\frac{S_{gr}}{S_g} + \frac{(1-E_p)}{E_p}]} - \frac{(p_i/Z_i)G_{pm}}{GE_p[\frac{S_{gr}}{S_g} + \frac{(1-E_p)}{E_p}]} \qquad (18)$$

In this form, it is clear that Eq. 18 expresses the end point, (p_m/Z_m), as a linear function of the ultimate gas recovery, and that the line passes through the point G, initial gas in place, at a zero value of (p_m/Z_m). This fact suggests a graphical solution of the water influx gas reservoir performance problem. If (p/Z) vs G_p under water drive can be estimated by any appropriate method (e.g., Eqs. 11 through 16 above), the intersection of the performance (p/Z) vs G_p plot and that of Eq. 18 represents the estimated ultimate gas recovery.

The water-drive gas reservoir performance calculation method presented above by Eqs. 11 through 16 is just one of a number of possible methods. See for example, Bruns, et al.,[11] Katz, et al.,[9] and Hubbard and Elenbass.[10]

The Carter-Tracy form of the water influx equation (Eq. 6) does deserve some comment. It is an approximate expression obtained from solution of a superposition equation by Laplace transform methods. Carter and Tracy present complete details of the derivation in Ref. 6. Their interpretation of the approximate nature of their water-influx expression appears to be too restrictive. On investigation, this method was found to be as accurate as any finite difference superposition method currently available. Restrictions concerning changes in water influx rate and the length of incremental time periods used appear to be no worse for this method than for normal superposition methods. This method does not assume that gas production rate should be constant. In the case of constant gas production rate, the water influx solution presented by Hurst[5] can be modified to describe gas reservoir behavior and is even more convenient to use, although the method has some restrictions in connection with the material balance. A brief description of the modification of Hurst's solution for application to water-drive gas reservoirs is given in the Appendix.

RESULTS OF PERFORMANCE PREDICTION

The prediction of unsteady-state performance will include: (1) matching past performance; (2) prediction of future performance and (3) deliverability.

Matching Past Performance

A number of investigators have analyzed the problem of estimating reserves initially present in a hydrocarbon reservoir from pressure-production performance. Operations involve matching past performance to determine system constants, then use of the constants to forecast the reservoir performance under a variety of field production schedules. Some performance history (production volumes and average field pressures as functions of time) showing a definite pressure decline is required. In the case of water-drive gas reservoirs, Eqs. 4 and 6, or Eqs. 4 and 10 can be used to determine the system constants. For example, if two reservoir pressures at two corresponding production points are available, at least in theory, two system unknowns can be determined. For water-drive gas reservoirs, this would include initial gas in place, G, and usually the water influx constant, B. Often, a very large number of production points are known. This can permit determination of other system constants such as the aquifer diffusivity, size, and geometry. In the general case, many more sets of data are available than are required by the number of unknowns. When the number of available equations exceeds the number of unknowns, the solution demands least-mean-square fitting techniques. For this, the rigorous material balance equation can be used in one of the following equivalent forms, or a combination thereof:

$$G_{pn}B_{gn} + B_w W_{pn} = G(B_{gn} - B_{gi}) + B \Sigma \Delta p_n Q_D(t_D) \qquad (19)$$

$$G_{pn}B_{gn} + B_w W_{pn} = G(B_{gn} - B_{gi}) + B \Sigma \Delta p_n Q_D(t_D) + C \qquad (20)$$

$$\frac{G_{pn}B_{gn} + B_w W_{pn}}{\Sigma \Delta p_n Q_D(t_D)} = G \frac{(B_{gn} - B_{gi})}{\Sigma \Delta p_n Q_D(t_D)} + B \qquad (21)$$

$$\frac{G_{pn}B_{gn} + B_w W_{pn}}{(B_{gn} - B_{gi})} = G + B \frac{\Sigma \Delta p_n Q_D(t_{Dn})}{(B_{gn} - B_{gi})} \qquad (22)$$

$$(G_{pn}B_{gn} + B_w W_{pn})(B_{gn} - B_{gi}) + G(B_{gn} - B_{gi})^2$$

$$+ B(\Sigma \Delta p_n Q_D(t_D))(B_{gn} - B_{gi}) \qquad (23)$$

$$(G_{pn}B_{gn} + B_wW_{pn})(B_{gn} - B_{gi}) = G(B_{gn} - B_{gi})^2$$

$$+ B(\Sigma\ \Delta p_n Q_D(t_D))(B_{gn} - B_{gi}) + C \qquad (24)$$

$$(G_{pn}B_{gn} + B_wW_{pn})(\Sigma\ \Delta p_n Q_D(t_D)) = G(B_{gn} - B_{gi})(\Sigma\ \Delta_{pn} Q_D(t_D))$$

$$+ B(\Sigma\ \Delta p_n Q_D(t_D))^2 \qquad (25)$$

$$(G_{pn}B_{gn} + B_wW_{pn})(\Sigma\ \Delta p_n Q_D(t_D)) = G(B_{gn} - B_{gi})(\Sigma\ \Delta p_n Q_D(t_D))$$

$$+ B(\Sigma\ \Delta p_n Q_D(t_D))^2 + C \qquad (26)$$

Eq. 19 is an obvious combination of Eqs. 4 and 10. Eqs. 20 through 26 are algebraic rearrangements of Eq. 19.

As can be seen, Eqs. 21 and 22 are in straight line form. The straight line method is commonly used for applying least-mean-square fitting techniques. This method was first recognized by van Everdingen, et al.,[23] and later adopted by various authors.[10,11,24] Application of this form to partial water-drive gas reservoirs has appeared recently. Chierici, et al.,[4] presented the results of studies of six actual gas fields and concluded that gas reserves of partial water-drive gas reservoirs could not be unequivocally determined from the past performance. This conclusion was derived from the principle of uncertainty which states that the internal structure of the system (the reservoir plus the aquifer) cannot be unequivocally determined from its external behavior. Variations in the computed values of reserves and water influx parameters have been attributed to random errors in determination of production rates and pressure data.

The material balance equation has been used for many years by engineers to predict oil reservoir performance. Several algebraic forms of the equation have been utilized to determine the system constants. Surprisingly, no study has been reported wherein various forms of the material balance equation were applied to the same reservoir, and the constants so obtained compared. A study is needed to check the validity of straight line form of equation, or find a suitable alternative form.

In order to investigate the effect of various equivalent forms of the material balance equation, a carefully-computed sample problem was selected for performance matching. Various portions of the pre-computed history were selected for matching, and least-mean-square techniques were used with the various forms of the material balance equation to determine the system constants. Dimensionless time constants used in matching analyses were calculated from the conditions selected for the problem.

Pressures at equal increments of time (at end of six month period) were computed as specified by van Everdingen *et al.*[23] The classic superposition method was used in the matching procedure. In addition two cases of aquifer permeability were used in studying the effect of pressure history. Detailed results are presented in the Appendix. It was found that the estimated values of the initial gas in place and water influx constant varied significantly with the form of the material balance equation, the aquifer permeability and amount of past history used in matching. Although not shown, results will also vary with the geometry of the aquifer, the magnitude of the increments of time and the way the pressures are selected for matching. Variations will also be obtained in value of the dimensionless time constant, estimated initially for matching, when per cent absolute deviation between calculated and observed reservoir pressure is minimized. In this process a computed set of the system constants, including gas in place G, and water influx constant B, are employed.

In regard to the form of the equation and the amount of past history used, it appears that the most commonly used straight line forms, Eqs. 21 and 22, are rather poor for performance matching, especially when available past history is limited. In the early life of a field when performance history is comparatively meager and proper values of system constants are most needed, the straight line method may give rather erroneous results. It can be seen that Eqs. 23 and 25 provide the best results. Eq. 25 gave the best estimate of the system constants.

Clearly, significant differences in results may arise from the computing techniques used in matching past history, even though equations which are apparently equivalent are used. This result appears due to minimizing different variable groups in the least-mean-square procedures. Thus, at least a portion of system constant variability may be a result of the computing methods used.

Prediction of Future Performance

Performance for a water-drive gas reservoir was computed for a reservoir of 5000 acres in area surrounded by an infinitely-large aquifer. Detailed reservoir and aquifer conditions selected are presented in Tables I and II. To permit evaluation of the importance of several key parameters, performance was estimated for a range of aquifer permeabilities, reservoir production rates, initial formation pressures, residual gas saturations, and water influx reservoir invasion efficiencies. Results of this

TABLE I

RESERVOIR AND AQUIFER PROPERTIES

Aquifer	
Average thickness, h	100 ft
Average porosity, ϕ	15 per cent BV
Average permeability, k_w	5-500 md*
Formation volume factor for water, B_w	1.02 res bbl/surface bbl
Viscosity of water, μ_w	0.35 cp
Total compressibility for aquifer, c_t	$6.25 \times 10^{-6} psi^{-1}$
Volumetric displacement efficiency, E_p	65-85 per cent*
Gas Reservoir	
Reservoir area, A	5000 acres
Radius of reservoir, r_w	8340 ft
Average reservoir thickness, h	100 ft
Initial water saturation, S_w	25 per cent PV
Residual gas saturation, S_{gr}	25-45 per cent PV*
Initial reservoir temperature, T	211^oF
Initial reservoir pressure, p_i	3000-7000 psia*
Base temperature for gas volumes, T_{sc}	60^oF
Base pressure for gas volumes, p_{sc}	14.7 psia

*Values varied in study to permit evaluation under a variety of conditions.

TABLE II

PHYSICAL PROPERTIES OF GAS

Gas gravity (to air)	0.65
Pseudo-critical temperature for gas, T_{pc}	$373^{o}R$
Pseudo-critical pressure for gas, p_{pc}	670 psia

Pressure, psia	Z
500	0.963
1000	0.933
1500	0.911
2000	0.897
2500	0.892
3000	0.900
3500	0.920
4000	0.946
5000	1.008
6000	1.083
7000	1.159

study provide a sufficient range of conditions to indicate the need for specific reservoir studies in particular cases.

In regard to the physical properties of the gas considered in this study, the gas law deviation factors, Z, and viscosities were taken from correlations as functions of the pseudo-reduced temperatures and pressures. (See Ref. 7, pages 20, 21 and 265.)

Fig. 2 presents p/Z vs cumulative gas produced for a field production rate of 30 MMscf/D calculated both by the classic superposition method, Eq. 19, and by the Carter-Tracy influx equation, Eqs. 4 and 6. Initial formation pressure was 5000 psia. The aquifer permeability was 5 md; residual gas saturation was 35 per cent of pore volume; and volumetric invasion efficiency was 85 per cent of bulk volume. It can be seen that there is excellent agreement between the two methods.

Figure 3 presents calculated p/Z vs cumulative gas produced for various gas production rates and initial formation pressures. The Carter-Tracy water influx equation was used in calculations. Other conditions selected were the same as those of Fig. 2. The dashed lines on Fig. 3 represent the performance of a volumetric gas reservoir with zero water influx. The characteristic departure of

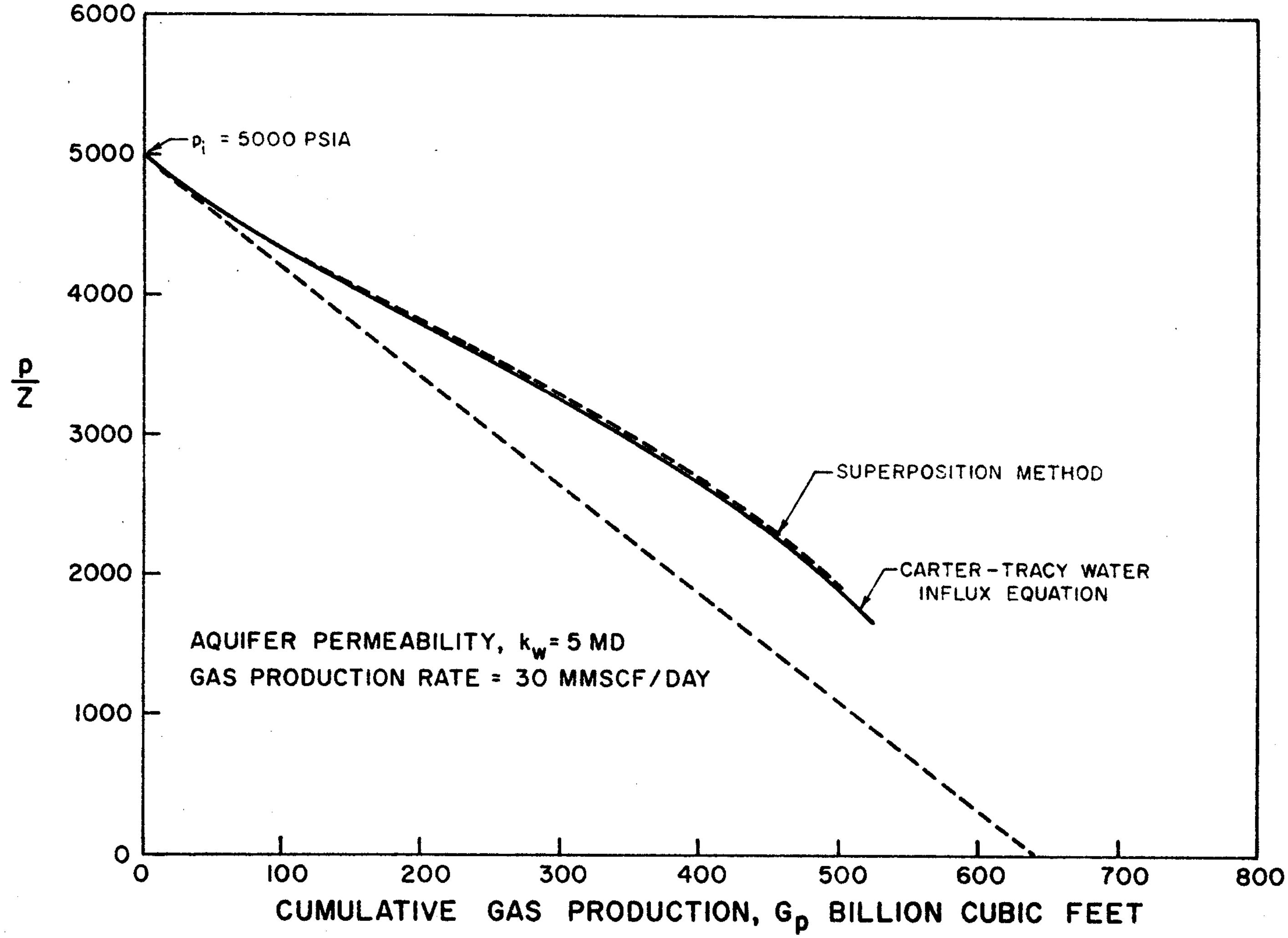

FIG. 2 - COMPARISON OF PERFORMANCE BY CLASSIC SUPERPOSITION AND USING THE CARTER-TRACY WATER-INFLUX EQUATION

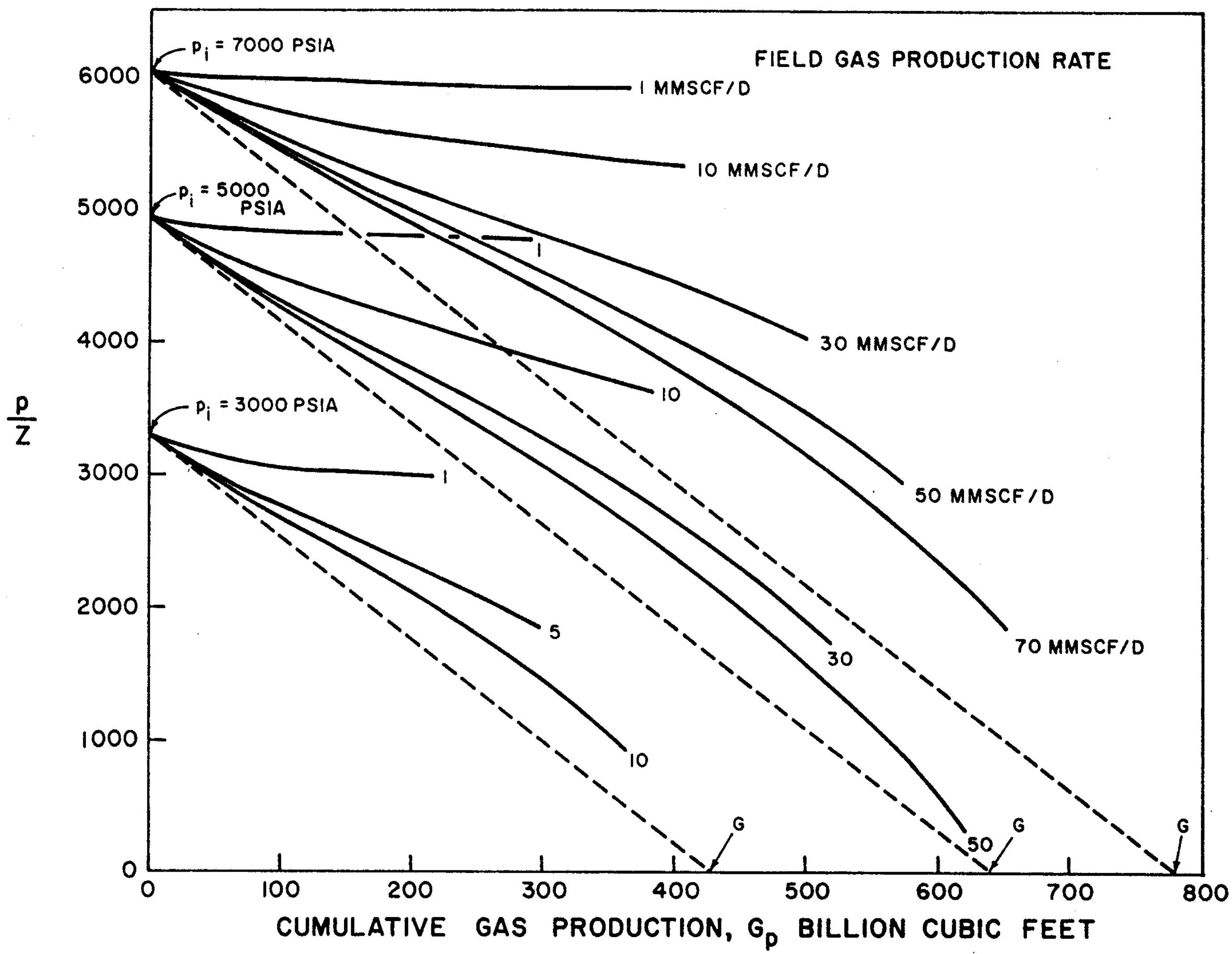

FIG. 3 - COMPUTED $\frac{p}{Z}$ VS. G_p FOR 5 MD CASE

water-drive gas reservoir performance from that of a volumetric gas reservoir where the p/Z curve has a linear relationship with G_p, is clearly shown. An important feature of the curves, however, is the effect of field production rate on the gas recovery. Each p/Z curve is carried to the end point dictated by the material balance of Eq. 17; the end point represents the estimated ultimate recovery. Clearly, gas recovery in the case of the water-drive gas reservoir may depend in a very important way upon production practices. A high field production rate permits production of an appreciable portion of gas by drawing down reservoir pressure before water influx completely engulfs the reservoir.

Figure 3 also indicates the effect of initial pressure level. This can be seen more clearly in Figures 4 and 5. In Fig. 4 the gas recovery as a per cent of initial gas in place is presented as a function of production rate for the 5 md permeability case and pressure levels of 3,000, 5,000, and 7,000 psia. Gas recovery tends to be lower at a given production rate for the high pressure reservoirs.

An inspection of Fig. 4 shows that gas recoveries range from about 45 per cent to in excess of 90 per cent of original gas in place, depending on the production rate. In regard to pressure level (see also Fig. 5) the

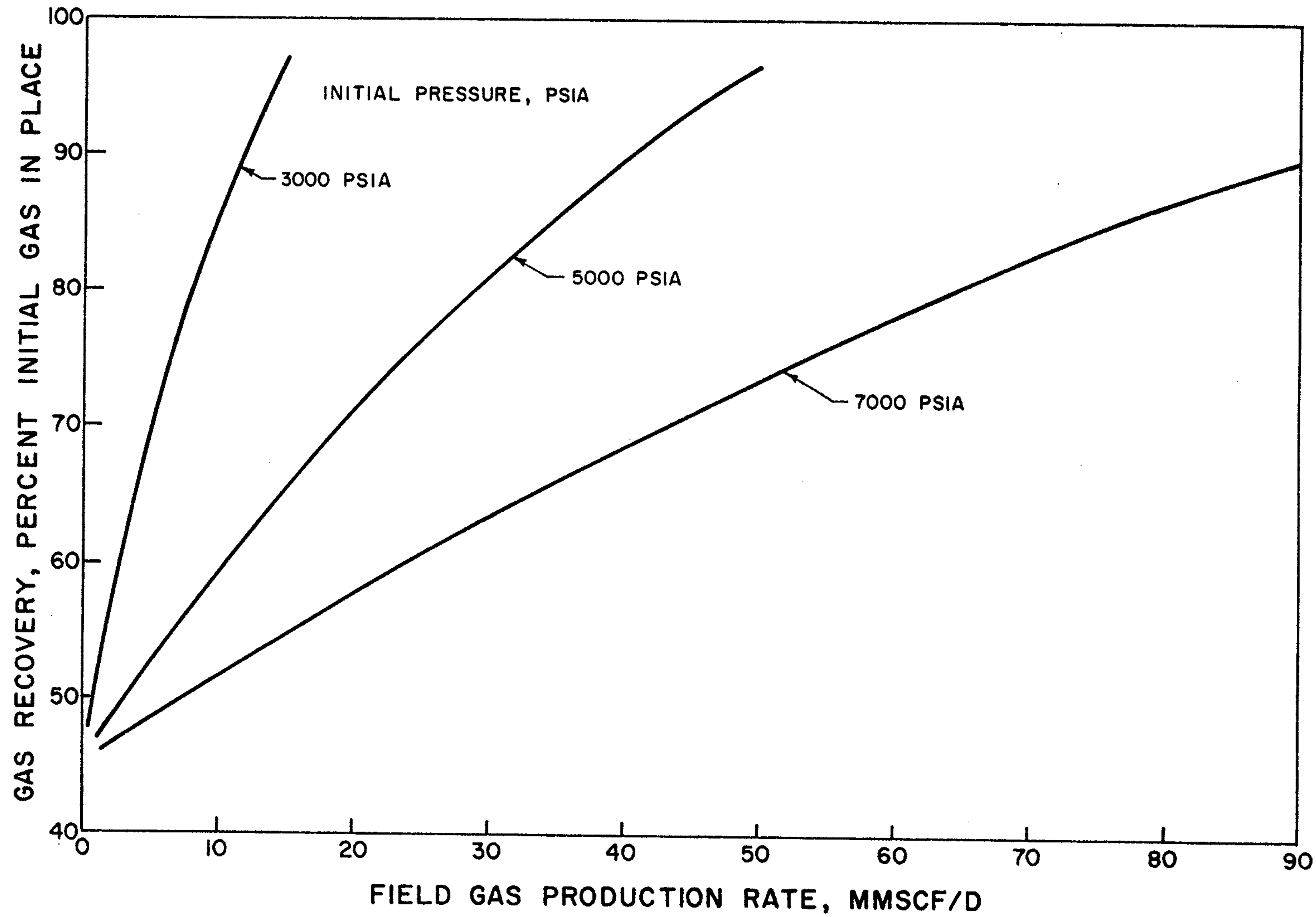

FIG. 4 – COMPUTED GAS RECOVERY VS. FIELD PRODUCTION RATE FOR VARIOUS INITIAL PRESSURES - 5 MD CASE

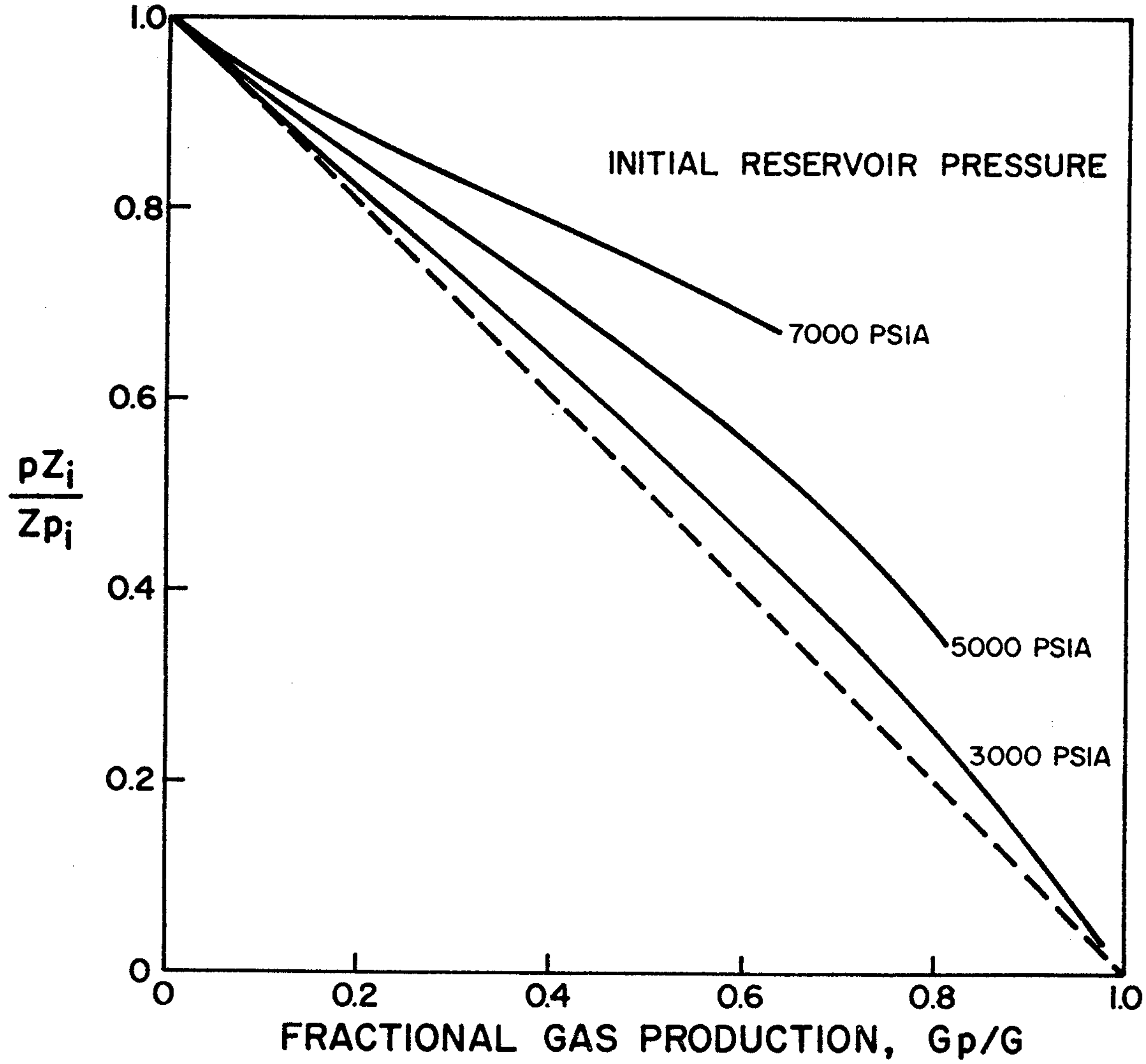

FIG. 5 - COMPUTED $(pZ_i)/(Zp_i)$ VS. Gp/G FOR 30 MMSCF/D AND 5 MD CASE

calculated gas recovery at a field production rate of 30 MMscf/D ranges from 63.5 per cent for an initial pressure of 7000 psia to in excess of 99 per cent for an initial pressure of 3000 psia.

Fig. 5 presents $\frac{p}{z} \cdot \frac{z_i}{p_i}$ vs gas recovery as a fraction of initial gas in place for a gas production rate of 30 MMscf/D with initial reservoir pressure as a parameter. This figure represents an alternate way to display the effect of initial pressure upon fractional gas recovery.

The effect of permeability upon gas recovery under water drive is shown in Figs. 6 and 7 for an initial pressure of 7000 psia. Gas recovery is less sensitive to production rate for practical production rates as aquifer permeability increases. Water influx responds so rapidly to pressure changes in the high permeability gas reservoir that it is not possible to benefit from increased production rate. In the limit, aquifer performance approaches a complete water drive as permeability increases. Note that computed gas recovery for the 500-md case shown in Fig. 7 ranges only from about 45 to 46 per cent for field production rates as high as 90 MMscf/D. Large errors can be made in estimating gas recovery for a water-drive gas reservoir from experience with volumetric gas reservoirs. The main factors in the complete water-drive gas reservoir are invasion efficiency and residual gas saturation.

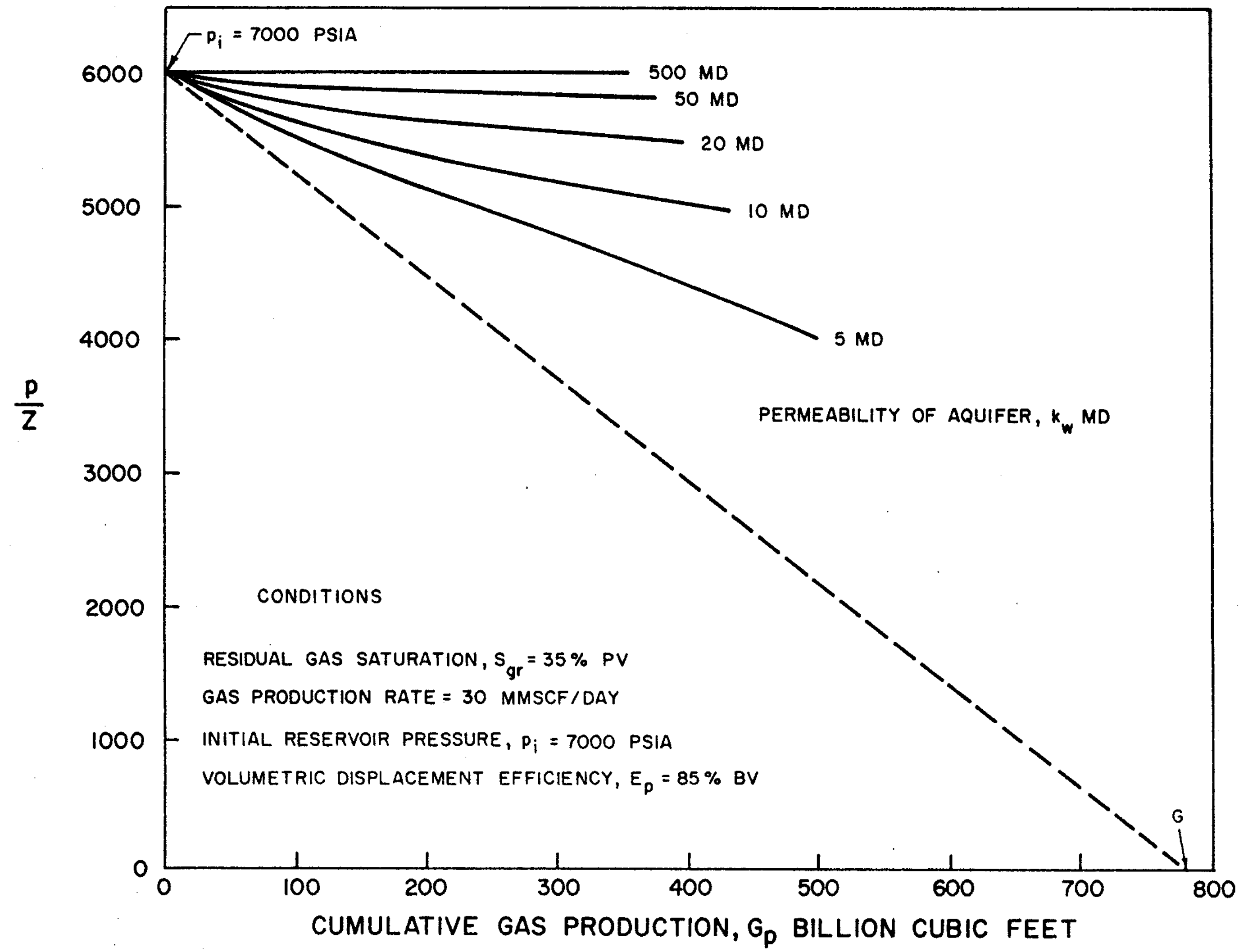

FIG. 6 - COMPUTED $\frac{p}{Z}$ VS. Gp FOR VARIOUS AQUIFER PERMEABILITIES

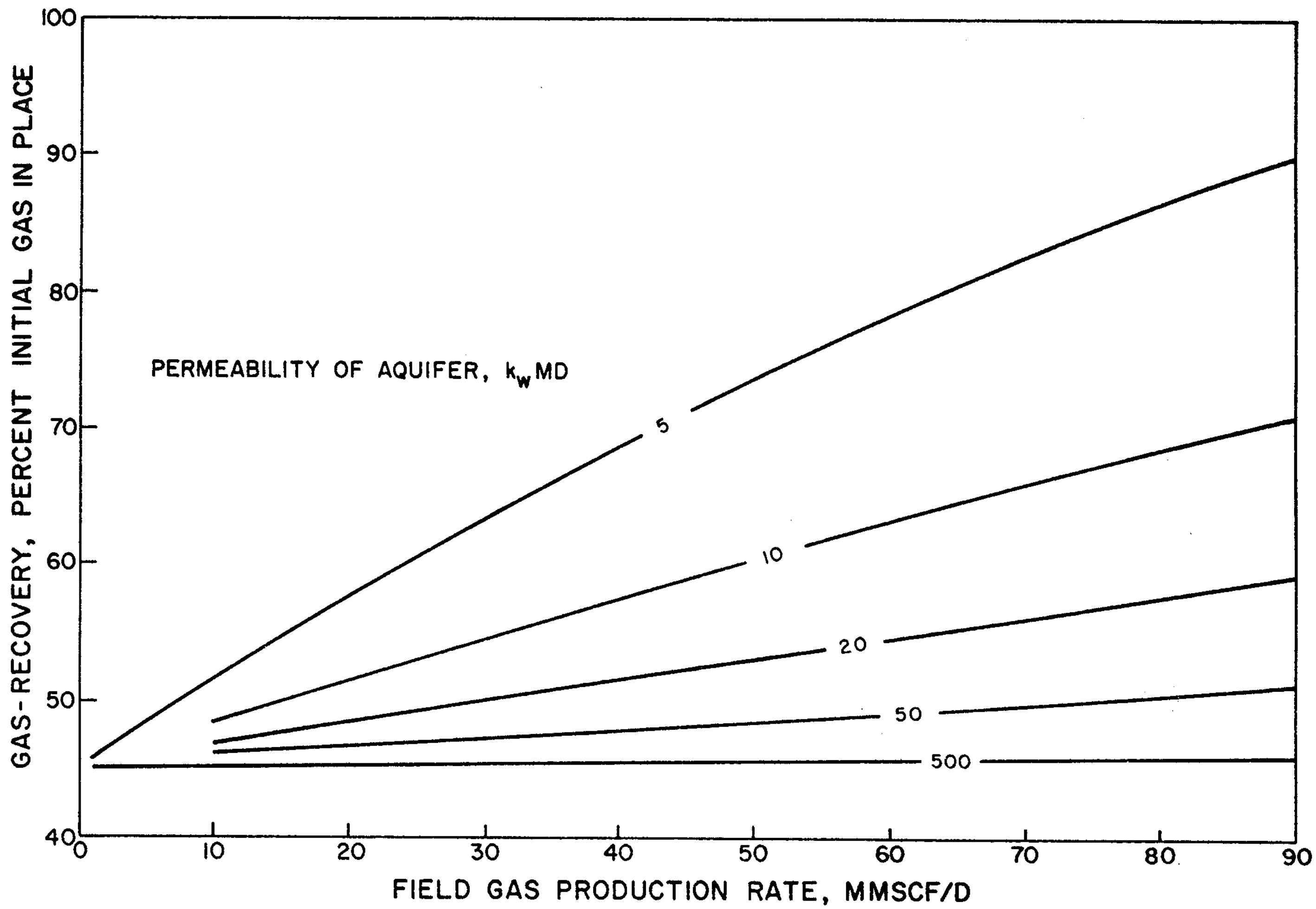

FIG. 7 - COMPUTED GAS RECOVERY VS. FIELD PRODUCTION RATE FOR VARIOUS PERMEABILITIES - INITIAL PRESSURE OF 7000 PSIA

Fig. 8 presents p/Z vs G_p curves for an initial pressure of 5000 psia and aquifer permeability of 5 md with the volumetric invasion efficiency, E_p, shown as a parameter. The residual gas saturation is 35 per cent of pore volume for all cases. This figure shows the effect of variation in volumetric displacement efficiency upon performance. The displacement mobility ratio is low, and thus displacement should be nearly piston-like. This has been shown experimentally by various investigators,[16-19] who show very little gas recovery after breakthrough under imbibition in linear models. It appears that the volumetric displacement efficiency should depend largely upon areal displacement efficiency, or well location, as long as it is assumed that the reservoir is homogeneous and water influx is from the edge, not the bottom of the reservoir. Water-coning could be a serious problem, and could place an upper limit on permissible well production rates if bottom-water invasion is encountered. Fig. 8 shows that gas recovery should depend somewhat on invasion efficiency but its importance should not be great if the well density is fairly uniform.

In order to cite quantitative gas recoveries, it is necessary to consider practical well deliverabilities and well spacings. If well spacing is 200 acres per well, 25 wells will be required for the 5000 acre reservoir used

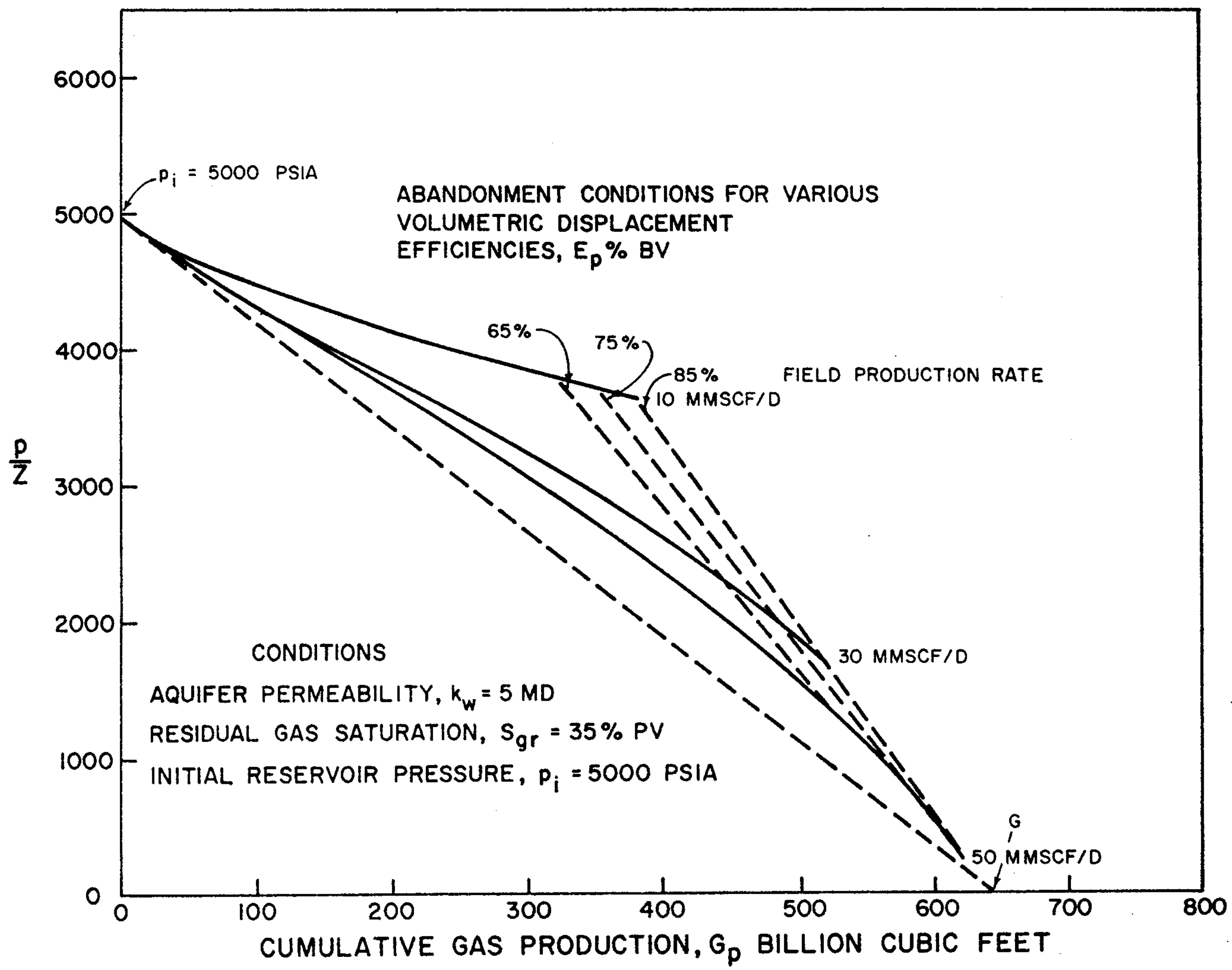

FIG. 8 - COMPUTED $\frac{p}{Z}$ VS. G_p FOR VARIOUS VOLUMETRIC DISPLACEMENT EFFICIENCIES

in the example. Thus field production rates of 10 to 70 MMscf/D would require per-well production rates of 400 Mscf/D to 2.8 MMscf/D. This range of production rates is reasonable for the example, as will be shown later.

Fig. 9 presents p/Z vs G_p for the 5000-psia, 5-md case and an invasion efficiency of 85 per cent of bulk volume, for various residual gas saturations. Residual gas saturation can have an important effect upon gas recovery at low production rates, as shown by Fig. 9. Clearly, accurate values of imbibition residual gas saturation are desirable for water-drive gas reservoirs performance estimation.

Another factor worth consideration in the water-drive gas reservoir is interruption of production. Shutting in gas production will result in reservoir pressure build-up. For the 5000-psia, 5-md case shown on Fig. 3, producing continuously at 30 MMscf/D or intermittently (on one year, off one year), the computed gas recovery decreased from 81.2 per cent of initial gas in place for continuous production to 66.4 per cent of initial gas in place for intermittent production. This was caused by an increase in abandonment pressure from 1,552 to 2,721 psia.

Some qualification is desirable here. The mechanism of gas entrapment by imbibition of water is not definitely

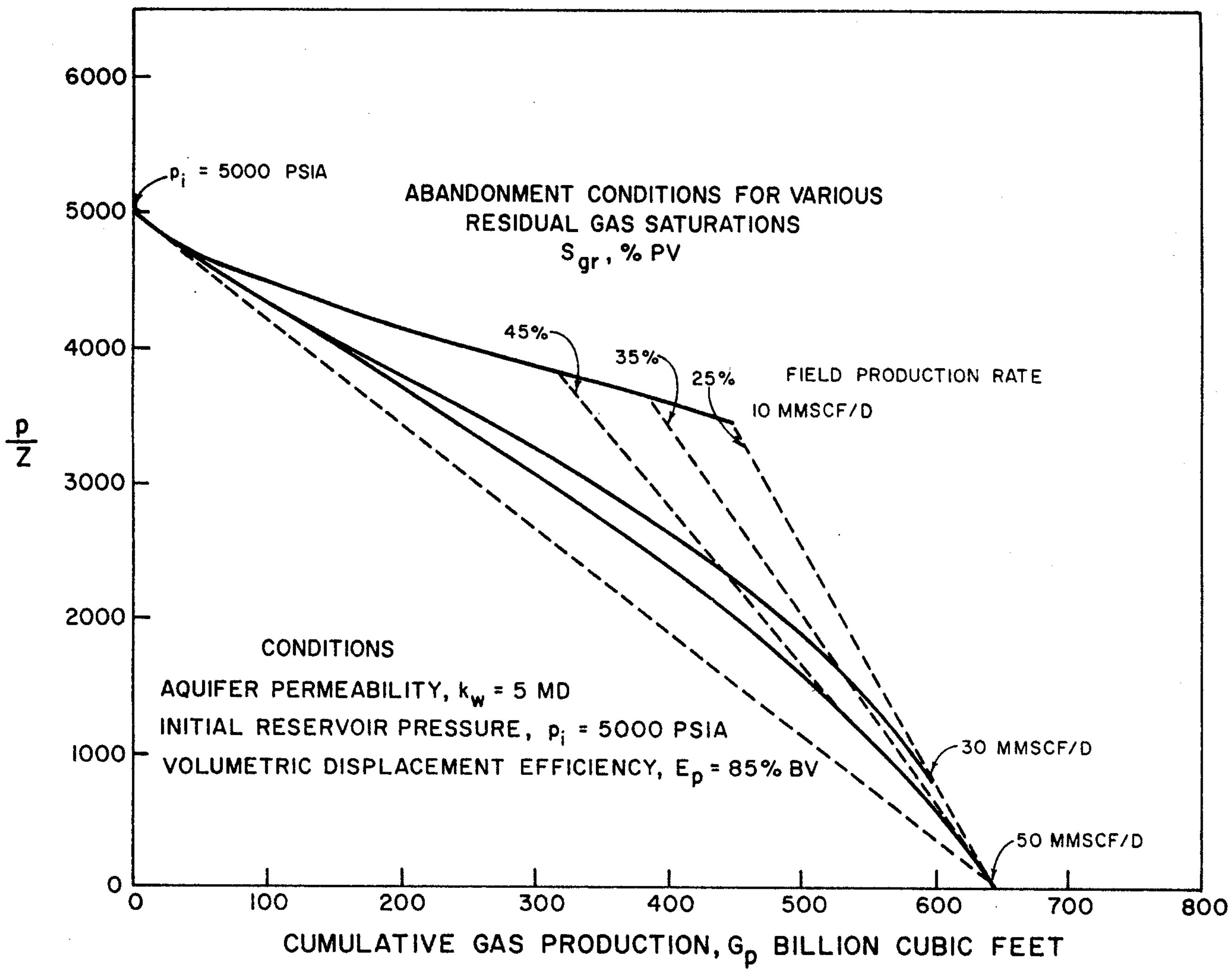

FIG. 9 – COMPUTED $\frac{p}{Z}$ VS. G_p FOR VARIOUS RESIDUAL GAS SATURATIONS

established. It has been speculated that: (1) gravity segregation[17] on interruption of production could lead to a reduction in the effective residual gas saturation; and (2) residual gas saturations may be reduced to very low values[13] under extended imbibition. Diffusion of gas through water has been cited, and often discredited for similar phenomena

Deliverability

Knowledge of well deliverabilities is required for proper interpretation of gas recoveries. That is, any selected field production rate implies a definite number of wells producing at reasonable rates. Gas flow can be expressed approximately by the modified Swift and Kiel[25] equation:

$$q_g = \frac{19.87 \times 10^{-6} k_g h T_{sc} (p_{ws}{}^2 - p_{wf}{}^2)}{\bar{\mu}_g p_{sc} T \, \bar{Z} \, [\ln \frac{r_d}{r_w'} + s + Dq_g]} \tag{27}$$

When gas flow becomes stabilized, the transient drainage radius r_d as defined by Aronofsky and Jenkins[26] becomes $0.472\ r_w$.* Eq. 27 can be used to produce

---

*The symbol r_w is normally used for "well radius" in well flow problems, but it is also taken as the "inner boundary" or reservoir radius in water influx reservoir studies. We use r_w to represent initial gas reservoir, or drainage area radius, while r_w' represents the well radius in Eq. 27.

back-pressure curves for any specific static pressure, p_{ws}, and well spacing (or r_w). Fig. 10 presents stabilized curves computed from Eq. 27 for a well spacing of one well per 200 acres. Formation parameters are listed in Table III. This method of producing stabilized curves is similar to one proposed by Carter, et al.[27] Similar curves can be produced for any well spacing by modifying the value of r_w.

Once deliverability information has been developed from the back pressure curves as a function of well spacing and static pressure, it is possible to determine the number of wells required to establish any desired field production rate. This information can usually be displayed as a plot of maximum field rate vs well spacing, with formation pressure level as a parameter.

In the case of water-drive gas reservoirs, water influx proceeds during the life of the field and well locations will become flooded-out. Hence, it is necessary to calculate the producible area at each stage of production to permit an estimate of the number of wells which are capable of production. This can be accomplished as follows, if we assume that a fraction of the total area of the field, $(1-E_p)$, cannot be drained by existing wells at abandonment--although some gas will be recovered

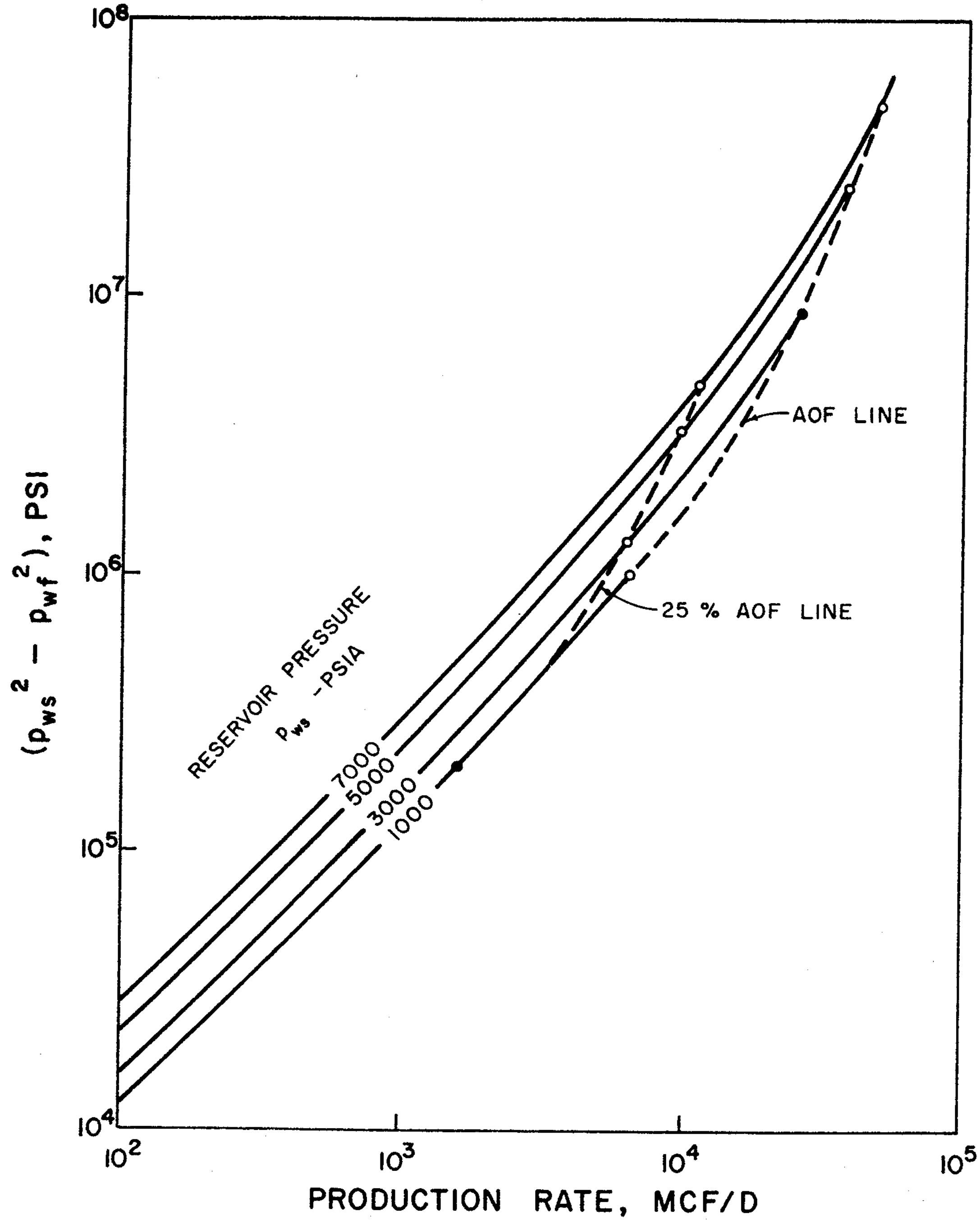

FIG. 10-COMPUTED STABILIZED DELIVERABILITY CURVE FOR WELL SPACING OF 200 ACRES

TABLE III

CONDITIONS SELECTED FOR EXAMPLE ESTIMATION OF STABILIZED BACK-PRESSURE CURVES

k_g	5 md
T_{sc}	520°R
p_{sc}	14.65 psia
s	-3
D	$2.5 \times 10^{-4} (Mscf/D)^{-1}$
r_w'	0.3 ft
h	100 ft
T	211°F
gas gravity	0.65 (to air)
T_{pc}	373°R
p_{pc}	670 psia

by pressure depletion till abandonment. The fraction of the remainder producible at any time can be estimated by a material balance, because the water influx at any time "n" can be calculated by Eq. 15.

The total reservoir volume which will be invaded by water at abandonment is:

$$\text{res bbl} = \frac{1}{5.615}[Ah\phi(1 - S_w - S_{gr})E_p] \qquad (28)$$

The reservoir volume invaded at time "n" is (W_{en}, res bbl). Thus the fraction of the producible reservoir area which can be produced at time "n" is:

$$F' = 1 - \frac{5.615\ W_{en}}{[Ah\phi(1 - S_w - S_{gr})E_p]} \qquad (29)$$

The fraction F' for any given time "n" can be multiplied by the original number of wells to estimate the number of producible wells remaining at time "n". If well allowables cannot be transferred from wet wells, the field production rate will have to decline as wells become flooded out, or more wells will have to be drilled at a reduced spacing. If spacing is maintained constant, the estimation of field performance outlined previously can be used to estimate recovery. The procedure will be to use Eq. 29 at the end of each time increment of production to determine the number of wells producing. This number will then be used to reduce the field production rate proportionately for the next time step.

Calculations can be made for various well spacings to permit economic evaluation of the effect of well spacing on profitability of development for any given gas field. In the case of volumetric gas reservoirs, gas recovery is usually affected only a few per cent by changes in well spacing. Well spacing can indicate a maximum in discounted present worth for a volumetric gas reservoir. However, this represents a type of acceleration project because gas recovery is usually not increased sufficiently to justify the drilling of additional wells. For certain water-drive gas reservoirs, this may not be true. For low permeability water-drive gas reservoirs, gas recovery can be doubled in some cases by a significant increase in field production rate. Increased field production rate implies closer spacing to provide adequate deliverability. Thus, well-spacing studies for some water-drive gas reservoirs can lead to a maximum in discounted present worth, which is influenced by a significant increase in gas recovery, as well as an increased income rate.

CORRELATION OF RESIDUAL GAS SATURATION

To investigate the possibility of developing a correlation between residual gas saturation and petrophysical characteristics of reservoir rocks, 320 experimental determinations of imbibition residual gas saturations were obtained from various published and unpublished sources. Porosity, absolute permeability, initial gas saturation and rock type were known for each determination. The distribution of samples provided a good representation of major rock types such as consolidated sandstones, limestones, unconsolidated sandstones and sands.

In view of the fundamental differences between the flow characteristics of consolidated sandstones, unconsolidated sandstones and sands, entirely different correlations by rock type were expected. See Naar, *et al.*,[28] and Wygal.[29]

To develop such correlations, data points were segregated by rock type. Four distinct sets of data were obtained. These include: (1) consolidated sandstones, (2) limestones, (3) unconsolidated sandstones, and (4) unconsolidated sands.

Multiple regression analysis techniques were applied to each set of data to develop suitable relationships between residual gas saturation and parameters such as porosity, permeability, and initial gas saturation. Variables such as S_{gr} and S_{gi} were expressed in per cent of pore volume, whereas porosity, ϕ, was used as per cent of bulk volume. In all cases permeability was in md. Methods outlined by Guthrie, et al.,[30] and Rodriguez[31] were followed, and analysis was carried out on a digital computer. Terms of the equation were changed in a stepwise manner so that the effect of each variable could be evaluated. Some of the variables were employed up to the second power and several combinations of the most significant variables were tried. Although it was not possible to develop a general correlation of high accuracy, several useful regression equations for the residual gas saturation were obtained.

In regard to the correlation of consolidated sandstones, S_{gr} was plotted as a function of $\frac{1}{2}S_{gi}$ and compared with the Naar-Henderson line, $S_{gr} = \frac{1}{2}S_{gi}$. Fig. 11 presents S_{gr} vs $\frac{1}{2}S_{gi}$ for all consolidated sandstones. The comparison of the Naar-Henderson line (solid line on Fig. 11) and other data points is clearly shown. Although Chierici's data show a slightly different trend, other sets of data agree with one another as well as the Naar-Henderson line. The correlation obtained for all consolidated sandstones is:

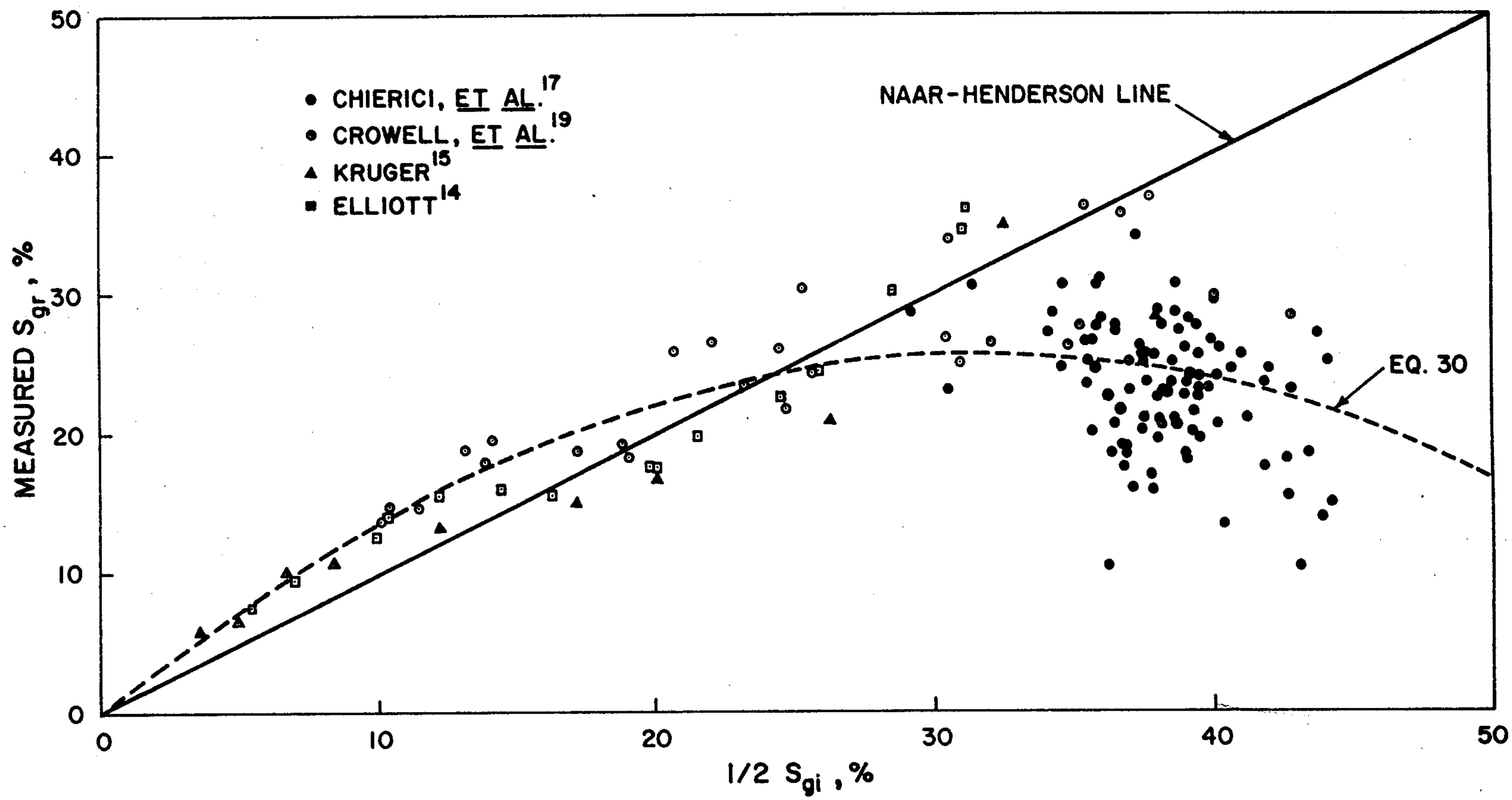

FIG. 11 - MEASURED S_{gr} VS. 1/2 S_{gi} FOR ALL CONSOLIDATED SANDSTONES

$$S_{gr} = A_1 S_{gi} + A_2 S_{gi}^2 \tag{30}$$

Coefficients of the above and the following regression equations are listed in Table IV. Ranges of variables employed in these correlations are shown in Table V.

Eq. 30 gave an average absolute deviation, $\bar{\sigma}_d$ of 14.15 per cent when calculated S_{gr} values were compared with S_{gr} measured. Absolute deviation, σ_d and average absolute deviation, $\bar{\sigma}_d$ are defined as:

$$\sigma_d = \frac{|S_{gr} \text{ measured} - S_{gr} \text{ calculated}|}{S_{gr} \text{ measured}} \times 100 \tag{31}$$

and

$$\bar{\sigma}_d = \frac{\sum_{i}^{n} \sigma_i}{n} , \tag{32}$$

where n is the total number of data points considered in a particular regression analysis. Distributions of per cent deviation as per cent of data points less than indicated deviation for all the correlations developed in this study are shown in Fig. 12. A comparison between the various regression equations can be clearly seen.

Fig. 13 presents S_{gr} calculated vs S_{gr} measured for <u>all</u> consolidated sandstone data. This type of figure represents a very good way to see just how good the

TABLE IV

COEFFICIENTS OF REGRESSION EQUATIONS

	A_1	A_2	A_3	A_4	A_5
Eq. 30	0.80841168	$-0.63869116 \times 10^{-2}$			
Eq. 33	-0.53482234	0.33555165×10	0.15458573	0.14403977×10^{2}	
Eq. 34	-0.51255987	$0.26097212 \times 10^{-1}$	-0.26769575	0.14796539×10^{2}	
Eq. 35	0.49361572×10	$-0.30044507 \times 10^{-1}$	$-0.20134031 \times 10^{-1}$	$0.16153951 \times 10^{-1}$	$-0.14482752 \times 10^{3}$

TABLE V

RANGE OF VARIABLES EMPLOYED IN CORRELATION OF RESIDUAL GAS SATURATION

Type of Rock	No. of Data	Variable	Range Maximum	Range Minimum
Consolidated Sandstones	153	ϕ	29.54	14.4
		k	2440.00	18.0
		S_{gi}	88.50	7.1
Limestones	86	ϕ	31.50	14.7
		k	470.00	3.5
		S_{gi}	79.40	47.3
Unconsolidated Sandstones	23	ϕ	34.00	28.0
		k	3720.00	750.0
		S_{gi}	62.00	10.3
Unconsolidated Sands	58	ϕ	52.40	24.6
		k	7700.00	42.0
		S_{gi}	84.00	62.0

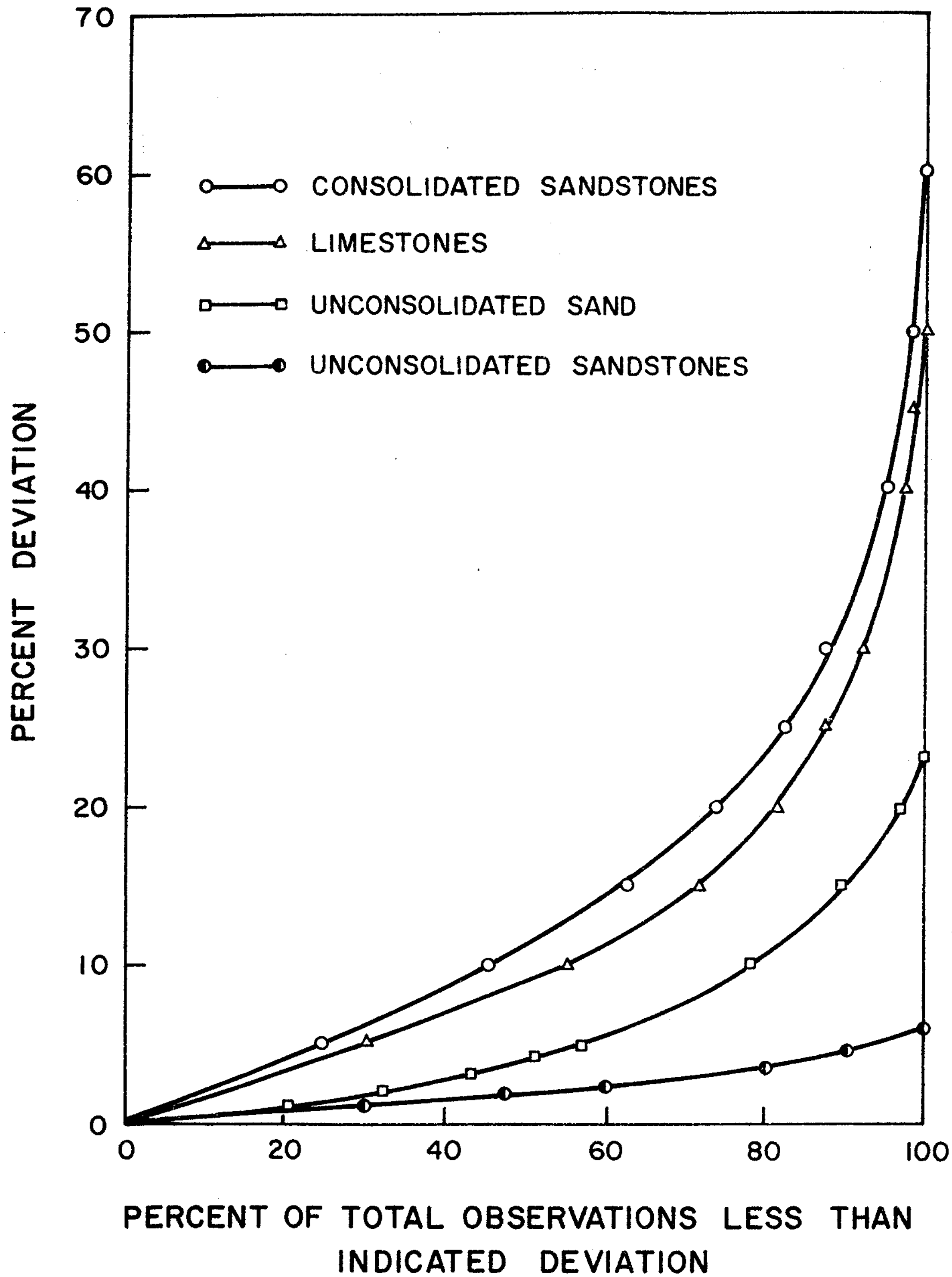

FIG. 12 - COMPARISON OF PERCENT DEVIATION FOR VARIOUS REGRESSION EQUATIONS

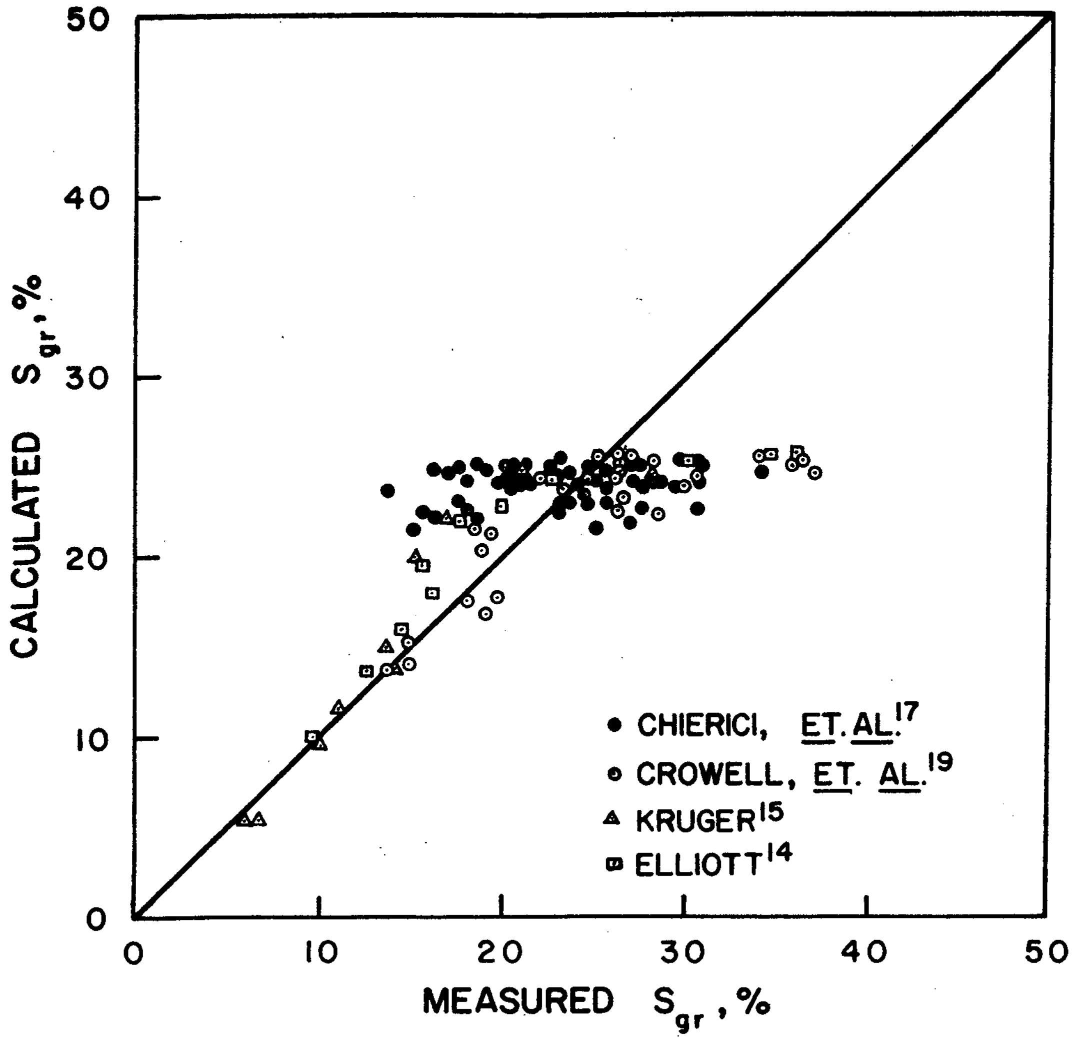

FIG. 13 — CALCULATED VS. MEASURED RESIDUAL GAS SATURATIONS — CONSOLIDATED SANDSTONES

correlation is. Here again the Chierici, et al. data, for apparently unknown reasons show a diversified trend.

The correlation obtained for the limestone data is:

$$S_{gr} = A_1\phi + A_2\log k + A_3 S_{gi} + A_4 \qquad (33)$$

Note that permeability is an important parameter in this correlation. Average absolute deviation in this case was 12.61 per cent. From the plot of S_{gr} calculated and S_{gr} measured in Fig. 14, it appears that the correlation is reasonable. Since the limestone samples were obtained from a single source, this correlation should be used with caution.

In regard to the correlations of unconsolidated sandstones and sands, S_{gr} was plotted as a function of S_{gi} as shown in Fig. 15. At first, it appeared that a single correlation might be possible for both unconsolidated sandstones and sands. Results of several correlation attempts indicated that one correlation would not be as good as separate correlations.

The correlation obtained for unconsolidated sandstones is:

$$S_{gr} = A_1 S_{gi} + A_2(S_{gi}\phi) + A_3\phi + A_4 \qquad (34)$$

whereas the correlation for unconsolidated sands is:

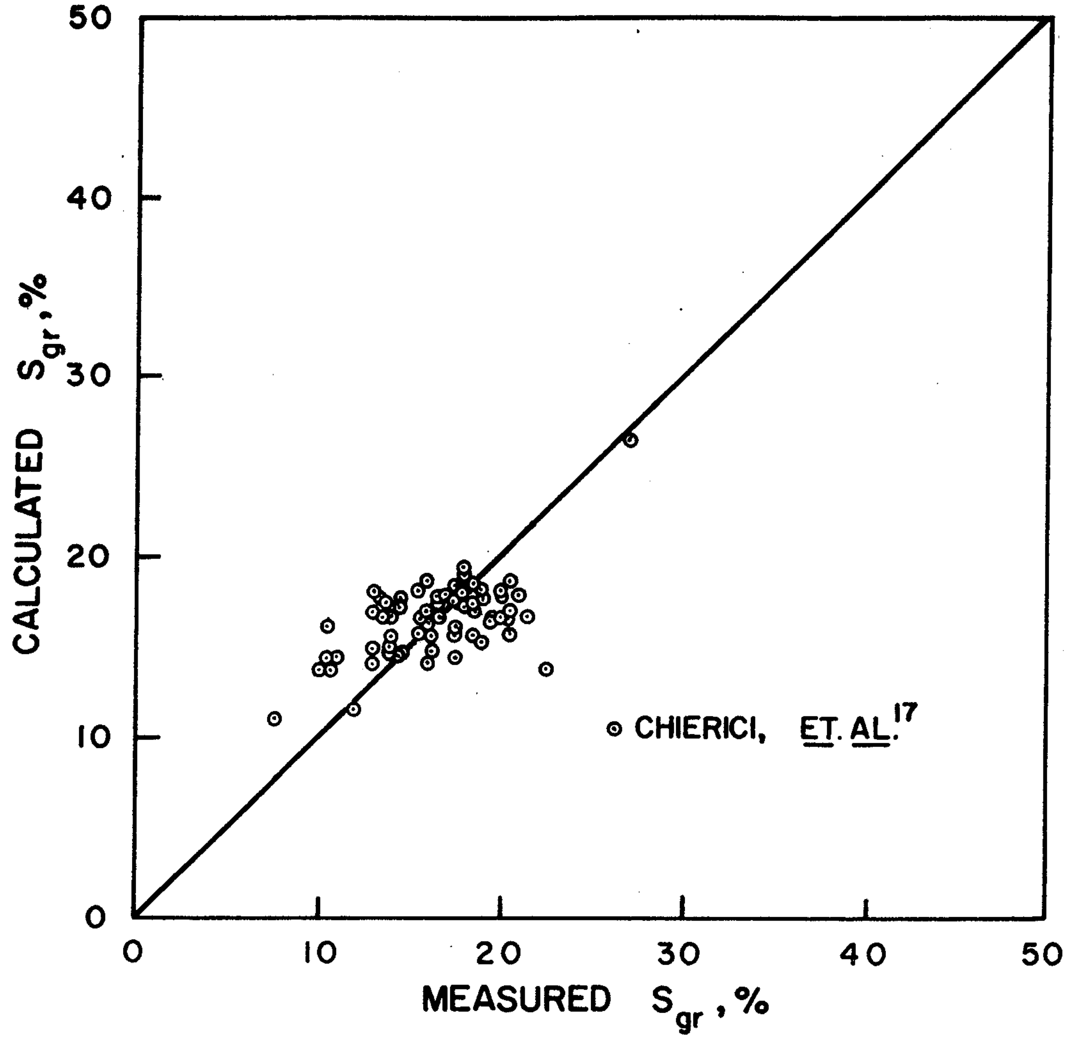

FIG. 14 — CALCULATED VS. MEASURED RESIDUAL GAS SATURATIONS — LIMESTONES

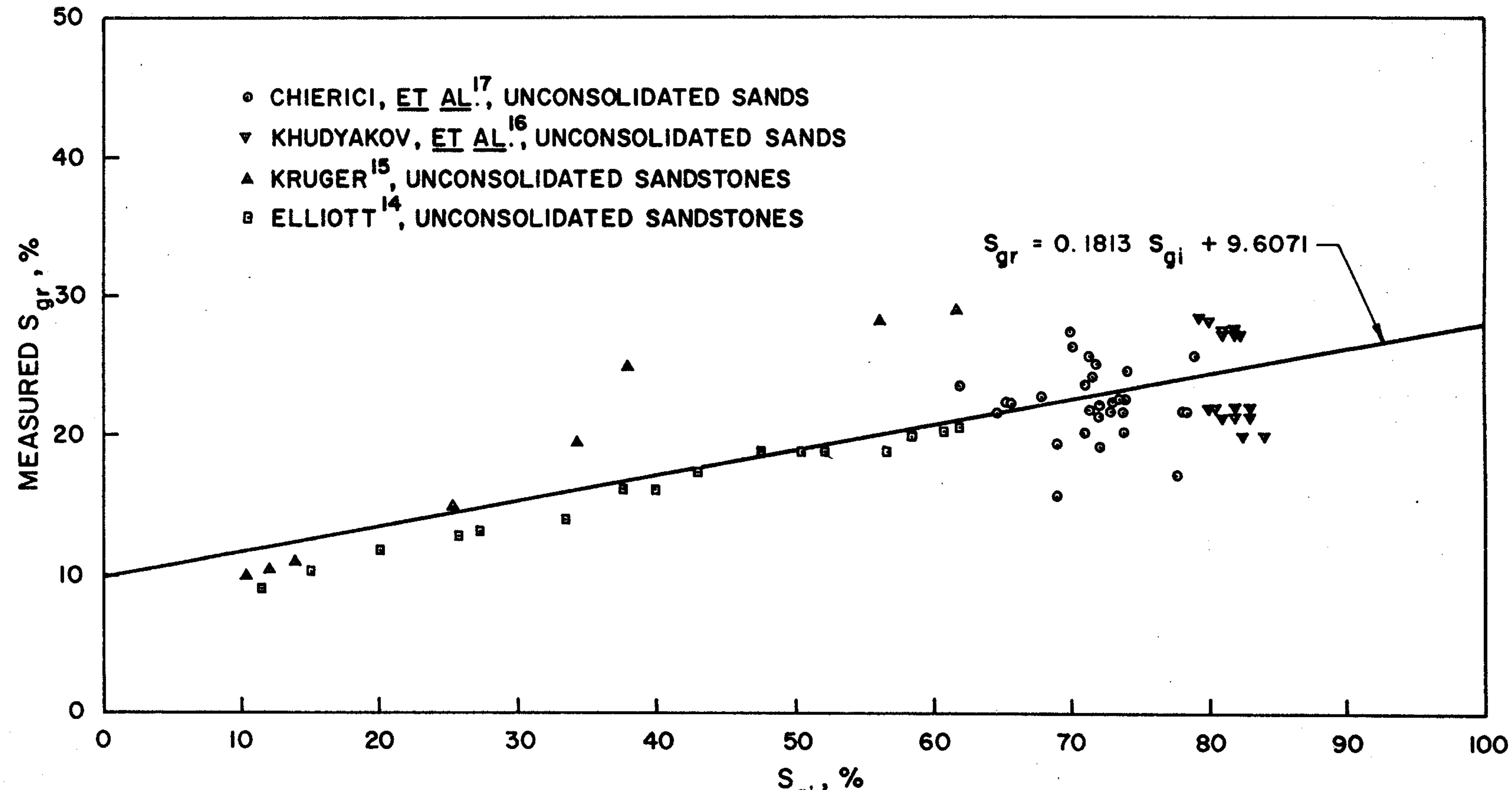

FIG. 15 - MEASURED S_{gr} VS. S_{gi} FOR UNCONSOLIDATED SANDSTONES AND SANDS

$$S_{gr} = A_1 S_{gi} + A_2 (S_{gi})^2 + A_3 (S_{gi} \phi) + A_4 \phi^2 + A_5 \qquad (35)$$

Eqs. 34 and 35 gave average absolute deviations of 2.33 and 5.54 per cent respectively. The plots of "S_{gr} calculated" and "S_{gr} measured" for unconsolidated sandstones and unconsolidated sands are shown in Figs. 16 and 17 respectively. It appears that the correlations are reasonably good. However, the number of data points is minimal for both correlations.

In view of the preceding, it appears that the regression equations developed for estimating the residual gas saturations are useful. Although the correlations are not of very high precision, these relationships should give reasonably good estimates of the residual gas saturation. When laboratory measurements are not available, S_{gr} values obtained from the above correlations should be used rather than an arbitrarily assumed value. Clearly, Eq. 30 for consolidated sandstones should provide better estimates of the residual gas saturation than the approximate Naar-Henderson expression, $S_{gr} = \frac{1}{2} S_{gi}$.

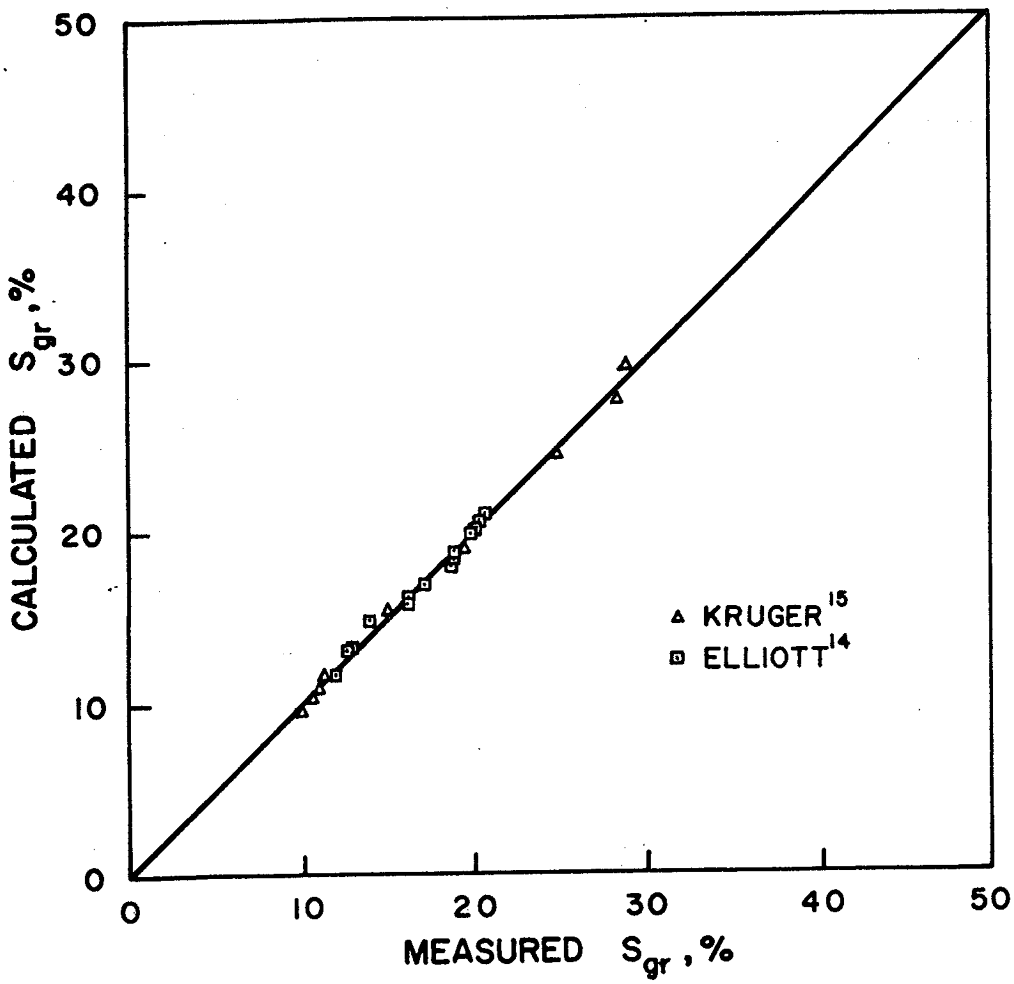

FIG. 16 – CALCULATED VS. MEASURED RESIDUAL GAS SATURATIONS – UNCONSOLIDATED SANDSTONES

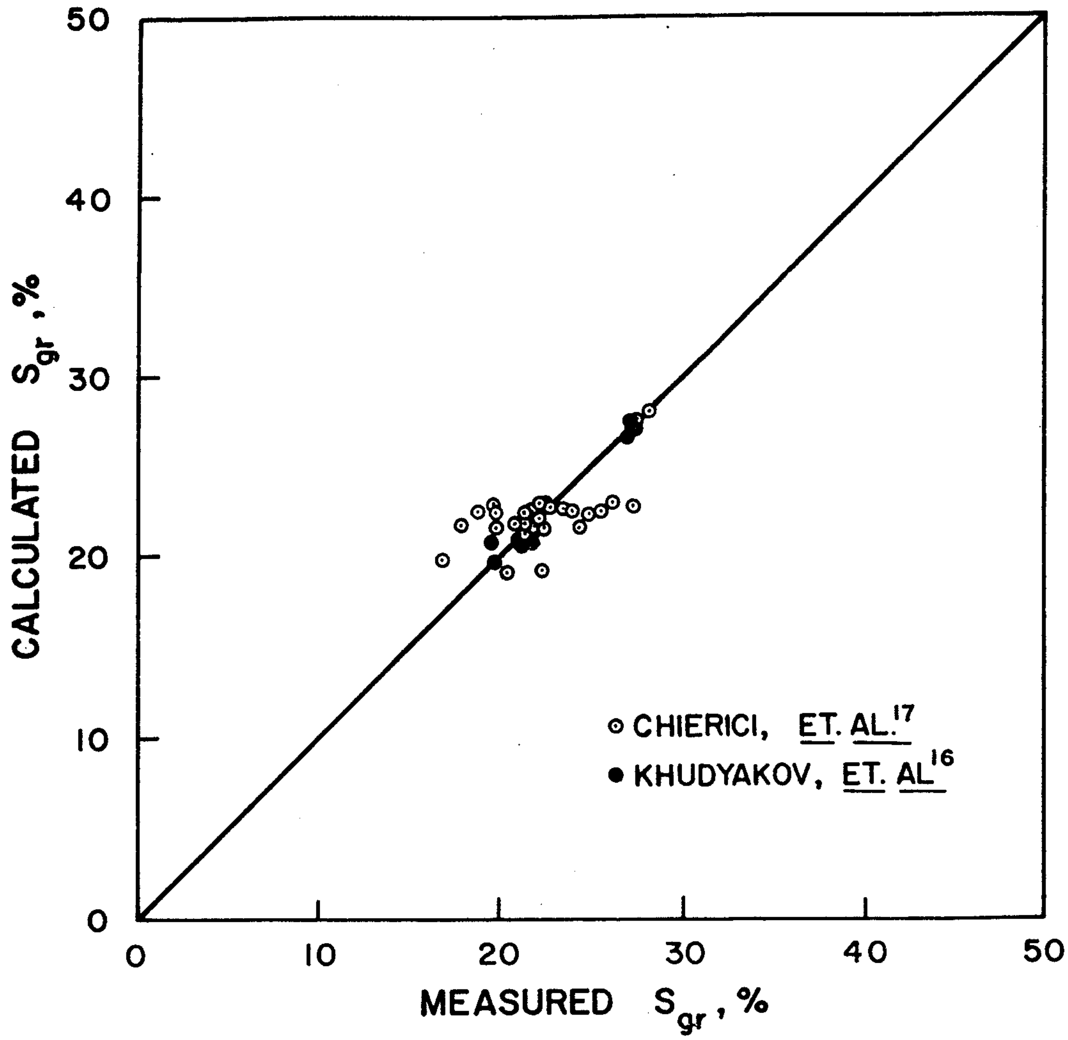

FIG. 17 — CALCULATED VS. MEASURED RESIDUAL GAS SATURATIONS — UNCONSOLIDATED SANDS

CONCLUSIONS

As a result of this study the following conclusions appear warranted:

The estimated values of the system constants (initial gas in place, water influx constant, etc.) by matching the past performance history depend upon various factors, some of which have not been generally recognized. These include the aquifer permeability, the form of the material balance equation, the geometry of aquifer, the magnitude of the increments of time and the way in which the Δp_n values are chosen.

In the early life of a field when past history is limited and accurate values of the system constants are most desirable, some forms (Eqs. 20 and 22) of the material balance equation may give unreliable estimate for the system constants and ultimate gas recovery. Nevertheless, results indicate that there is excellent agreement between actual performance and calculated performance in almost all the cases. It also appears that estimated ultimate recovery and performance match from the straight line method, Eq. 21, may be reasonably good even using the poor system constants. It does not appear desirable to obtain good answers with incorrect data, however. An alternate form of the material balance equation (Eq. 25) has been obtained which, under similar conditions, should give much better estimate of the system constants. Hence, Eq. 25 should provide the best results so far as estimated ultimate recovery and future reservoir performance of a water-drive gas reservoir are concerned. The above should be true even when only limited past history of a particular field is available. In any case, it is evident that at least a portion of system constant variability may be a result of computing techniques used in matching past history--even though equations which are apparently equivalent are used.

The system constants mainly considered in this study are the initial gas in place and the water-influx constant. Although not shown, variations in other system constants such as the dimensionless time and sometimes the size and approximate geometry of the aquifer may also be considered.

If it is assumed that the reservoir is reasonably homogeneous, and the water influx will be from the edge of the formation, it appears that gas recovery from certain water-drive gas reservoirs may be very sensitive to field production rate. If practical, the field should be produced at as high a rate as possible, and field curtailment should be avoided. This may result in a significant increase in gas reserves by lowering the abandonment pressure. However, it should be pointed out that the information on the effect of gas production rate on reserves, and the abandonment point presented in this study, depend on the assumption that water-coning is not the controlling factor. It is recognized that water-coning and reservoir heterogeneities often may be controlling. Water-coning may be a serious problem, and may place an upper limit on permissible production rate if bottom water is encountered.

The sensitivity of gas recovery to field production rate appears to decrease with increase in aquifer permeability and initial formation pressure. The information presented in this study should provide an indication as to when detailed studies might be worthwhile for particular reservoirs.

Computations indicate that gas recovery should depend somewhat on volumetric displacement efficiency, but the effect should not be great if the well density is uniform.

Residual gas saturation can have an important effect upon gas recovery, particularly at low production rates. An accurate determination of imbibition residual gas saturation is desirable for estimating reserves for water-drive gas reservoirs. Such a determination may be made in the laboratory. When laboratory measurements are not available, correlations developed in this study may be used to estimate residual gas saturation from reservoir rock characteristics which are generally available.

Estimates of gas recovery for water-drive gas reservoirs based on experience from volumetric gas reservoirs may be quite erroneous. If proper steps are not taken in the early life of the field, the ultimate gas recovery for a water-drive reservoir may be extremely low.

Finally it is emphasized that the potential importance of water influx in gas reservoirs likely to be under water drive should be investigated as early as possible so that adequate planning to optimize gas reserves can be made.

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NOMENCLATURE

A	=	area of reservoir
b	=	width of the reservoir rock in linear model, ft (see Table VI)
B	=	water influx constant, res bbl/psi (see Eq. 8)
B_{gi}	=	gas formation volume factor at pressure p_i, res bbl/scf
B_{gm}	=	gas formation volume factor at pressure p_m, res bbl/scf
B_{gn}	=	gas formation volume factor at pressure p_n, res bbl/scf
B_w	=	water formation volume factor, res bbl/ surface bbl
C	=	constant (see Eq. 20)
$\bar{c}_g$	=	average gas compressibility, psi^{-1}
C_n	=	function of t_{Dn} (see Eq. 12)
c_t	=	total compressibility for the aquifer, psi^{-1}
D	=	non-Darcy flow constant, $(Mscf/D)^{-1}$
d	=	differential operator
E_n	=	function of t_{Dn} (see Eq. 14)
E_p	=	volumetric invasion efficiency, fraction of bulk volume
F	=	correction for limited portion of reservoir perimeter open to water influx, fraction

F' = fraction (see Eq. 29)

G = original gas in place, scf

G_{pm} = cumulative gas produced at abandonment, scf

G_{pn} = cumulative gas produced at time t_n, scf

h = net thickness, ft

K = constant (see Eq. 13)

k = absolute permeability, md

k_g = effective permeability to gas in the reservoir, md

k_w = effective permeability to water in the aquifer, md

$p_D(t_{Dn})$ = van Everdingen-Hurst dimensionless pressure drop

$p_D'(t_{Dn})$ = first derivative with respect to t_{Dn} of $p_D(t_{Dn})$

p_i = initial formation pressure, psia

p_m = pressure at abandonment, psia

p_n = pressure at time t_n, psia

p_{sc} = standard condition pressure, psia

p_{pc} = pseudo-critical pressure, psia

p_{pr} = pseudo-reduced pressure, fraction

p_{wf} = bottom-hole flowing pressure, psia

p_{ws} = static reservoir pressure, psia

$Q_D(t_D)$ = dimensionless cumulative influx at dimensionless time, t_{Dn}

q_g = gas production rate, Mscf/D

R = gas law constant, 10.73 psia-cu ft/lb mol-^{o}R

r_d = transient drainage radius of Aronofsky and Jenkins, ft

r_w = reservoir radius, ft (radius of circle of area A)

r'_w = well radius, ft

s = skin effect, dimensionless

S_{gi} = initial gas saturation, fraction

S_{gr} = residual gas saturation, fraction

S_w = initial water saturation in gas reservoir, fraction

T = reservoir temperature, ^{o}R

t_{Dn} = dimensionless time at time t_n (see Eq. 7)

t_n = time, years

T_{pc} = pseudo-critical temperature, ^{o}R

T_{pr} = pseudo-reduced temperature, fraction

T_{sc} = standard condition temperature, ^{o}R

W_{en} = cumulative water influx at time t_n, res bbl

W_{pn} = cumulative water produced at time t_n, surface bbl

$\overline{Z}$ = real gas law deviation factor at average pressure (evaluated as per Ref. 27)

Z_m = deviation factor at abandonment pressure p_m

Z_n = deviation factor at pressure p_n

ϕ = porosity, fraction of bulk volume

$\overline{\mu}_g$ = average gas viscosity, cp (evaluated as per Ref. 27)

μ_w = viscosity of water, cp

σ_d = absolute deviation, per cent (see Eq. 31)

$\overline{\sigma}_d$ = average absolute deviation, per cent (see Eq. 32)

APPENDIX

AQUIFER FLOW GEOMETRIES

Several aquifer geometries have been considered and used in petroleum reservoir engineering. The most common of these are: radial, linear and hemispherical.

Radial Flow Model

This case has been extensively studied, and widely used. van Everdingen and Hurst[3] were the first to present results for finite and infinite radial systems. Edwardson, *et al.*,[21] have published high precision curve fit equations for an infinite radial aquifer.

Before the usual transient flow superposition equation for water influx, Eq. 10 in the main text, can be used, it is necessary to specify the dimensionless cumulative influx term, $Q_D(t_D)$. In general, this term depends upon the flow geometry, rock and fluid properties and time. $Q_D(t_D)$ for an infinite radial case can be computed from:[21]

$$Q_D(t_D) = 2\sqrt{\frac{t_D}{\pi}}, \qquad t_D \leq 0.01 \qquad \text{(A-1)}$$

$$Q_D(t_D) = \frac{1.2838\sqrt{t_D} + 1.19328t_D + 0.269872t_D^{3/2} + 0.00855294t_D^{2}}{1 + 0.616599\sqrt{t_D} + 0.0413008t_D}$$

$$0.01 < t_D < 200, \qquad \pm 0.02\% \qquad \text{(A-2)}$$

$$Q_D(t_D) = \frac{-4.29881 + 2.02566t_D}{\ln t_D} , \qquad t_D \geq 200 ,$$

$$\pm 0.07 \qquad \text{(A-3)}$$

To use the Carter-Tracy influx equation, Eq. 6 in the main text, it is necessary to specify the dimensionless pressure, $p_D(t_D)$, and its first derivative, $p'_D(t_D)$. For an infinite radial aquifer:[21]

$$p_D(t_D) = 2\sqrt{\frac{t_D}{\pi}} , \qquad t_D \leq 0.01 \qquad \text{(A-4)}$$

$$p_D(t_D) = \frac{370.529\sqrt{t_D} + 137.582t_D + 5.69549t_D\sqrt{t_D}}{328.834 + 265.488\sqrt{t_D} + 45.2157t_D + t_D\sqrt{t_D}} ,$$

$$0.01 < t_D < 500 , \qquad \pm 0.08\% \qquad \text{(A-5)}$$

$$p_D(t_D) = [\tfrac{1}{2}\ln t_D + 0.40454][1 + \frac{1}{2t_D}] + \frac{1}{4t_D} ,$$

$$t_D \geq 500 , \qquad \pm 0.01\% \qquad \text{(A-6)}$$

and

$$p'_D(t_D) = \frac{1}{\sqrt{\pi t_D}} , \qquad t_D \leq 0.01 \qquad \text{(A-7)}$$

$$p'_D(t_D) = \frac{716.441 + 46.7984\sqrt{t_D} + 270.038t_D + 71.0098t_D\sqrt{t_D}}{1269.86\sqrt{t_D} + 1204.73t_D + 618.618t_D\sqrt{t_D} + 538.072t_D^2 + 142.41t_D^2\sqrt{t_D}} ,$$

$$0.01 < t_D < 500 , \qquad \pm 0.08\% \qquad \text{(A-8)}$$

$$p'_D(t_D) = \frac{1}{2t_D} - \frac{1}{2t_D^2}[\tfrac{1}{2}\ln t_D + 0.40454] ,$$

$$t_D \geq 500 , \qquad \pm 0.05\% \qquad \text{(A-9)}$$

The above equations are particularly useful for digital computer solution of transient flow problems. $p'_D(t_D)$, for an infinite radial aquifer was presented previously in Fig. 1. In the case of finite aquifers, the van Everdingen Hurst solutions can be differentiated for times when the boundary effect is felt. $p'_D(t_D)$ will approach a constant value in this case.

Dimensionless groups involved in linear and hemispherical flow geometries have been obtained,[32] and are presented in Table VI. Fig. 18 presents $Q_D(t_D)$ vs t_D for hemispherical, radial and linear models. $p_D(t_D)$ and $p'_D(t_D)$ for various geometries are shown in Fig. 19 and 20, respectively.

TABLE VI

SUMMARY OF DIMENSIONLESS GROUPS FOR LINEAR AND HEMISPHERICAL MODELS

Dimensionless Group	Linear Model	Hemispherical Model
t_{Dn}	$\frac{2.31k_w t_n}{\phi\mu c_t}$	$\frac{2.31k_w t_n}{\phi\mu c_t r_w^2}$
$Q_D(t_{Dn})$	$2\sqrt{\frac{t_{Dn}}{\pi}}$	$t_{Dn}[1 + \frac{2}{\sqrt{\pi t_{Dn}}}]$
$p_D(t_{Dn})$	$2\sqrt{\frac{t_{Dn}}{\pi}}$	$1 - e^{t_{Dn}}[erfc(t_{Dn})]$
$p'_D(t_{Dn})$	$\frac{1}{\sqrt{\pi t_{Dn}}}$	$\frac{1}{\sqrt{\pi t_D}} - e^{t_{Dn}}[erfc(t_{Dn})]$
B	$0.1781bhc_t$	$1.1191\phi c_t r_w^3$

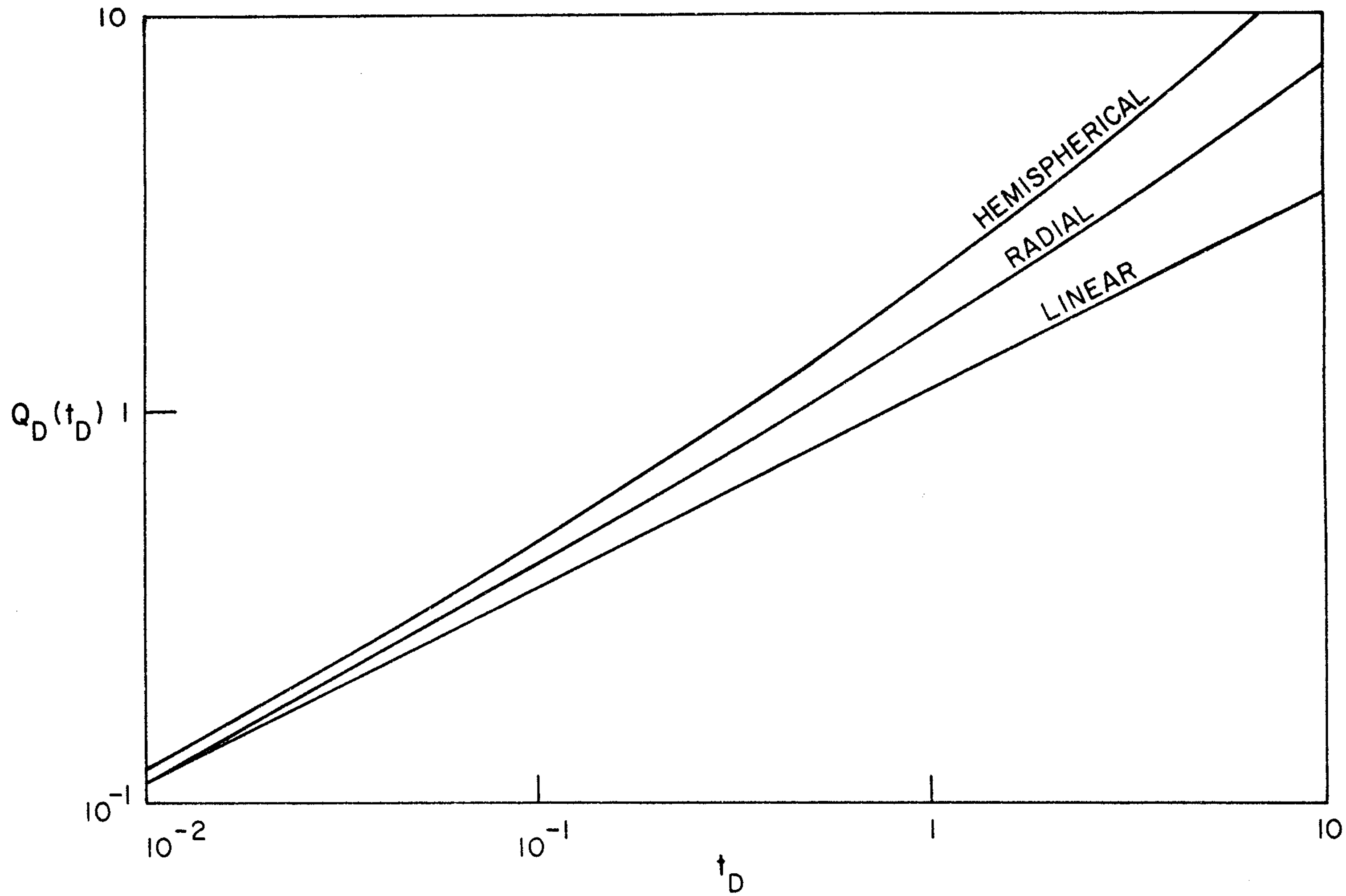

FIG. 18 - $Q_D(t_D)$ VS. t_D FOR VARIOUS AQUIFER GEOMETRIES (INFINITE CASE)

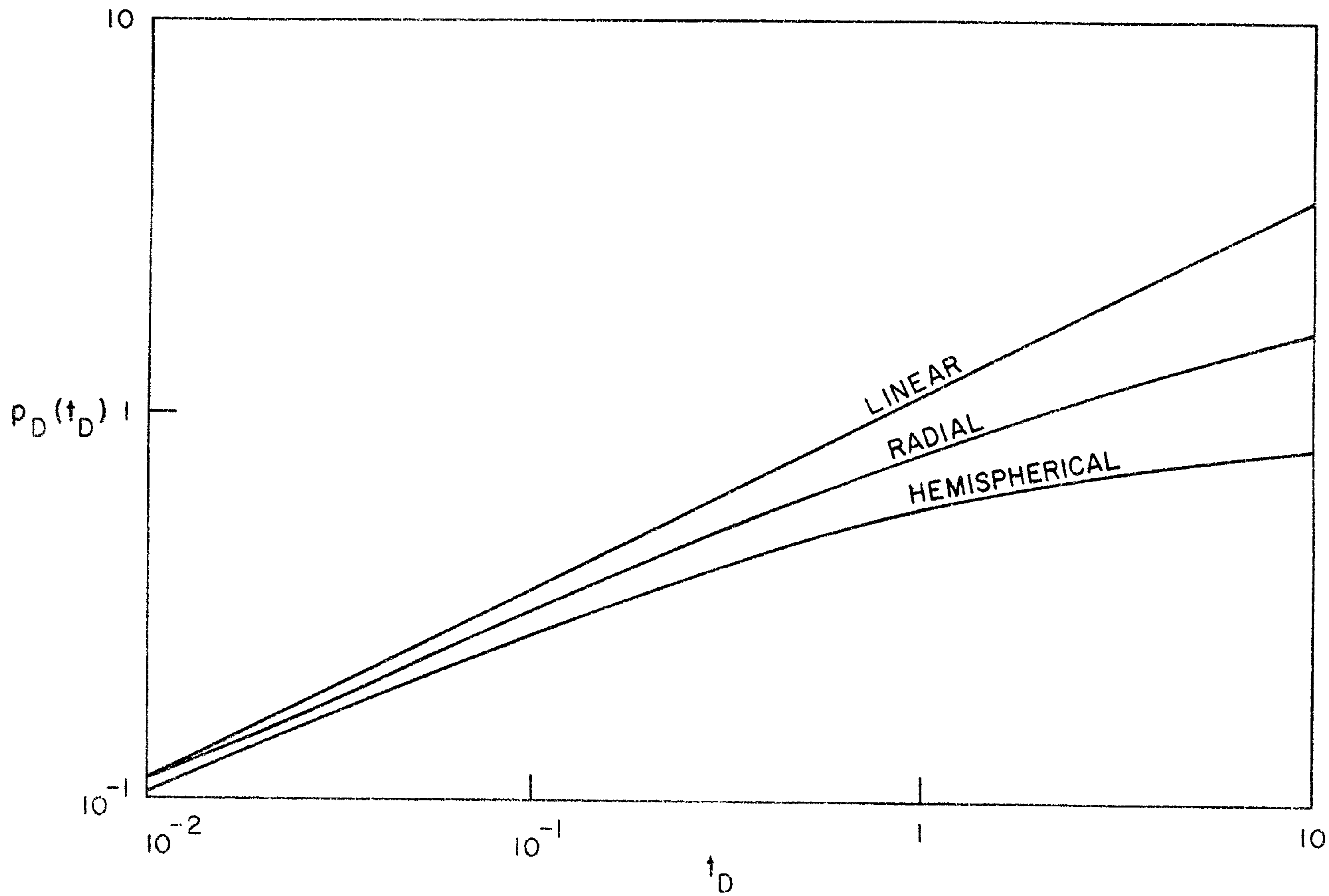

FIG. 19 – $p_D(t_D)$ VS. t_D FOR VARIOUS AQUIFER GEOMETRIES (INFINITE CASE)

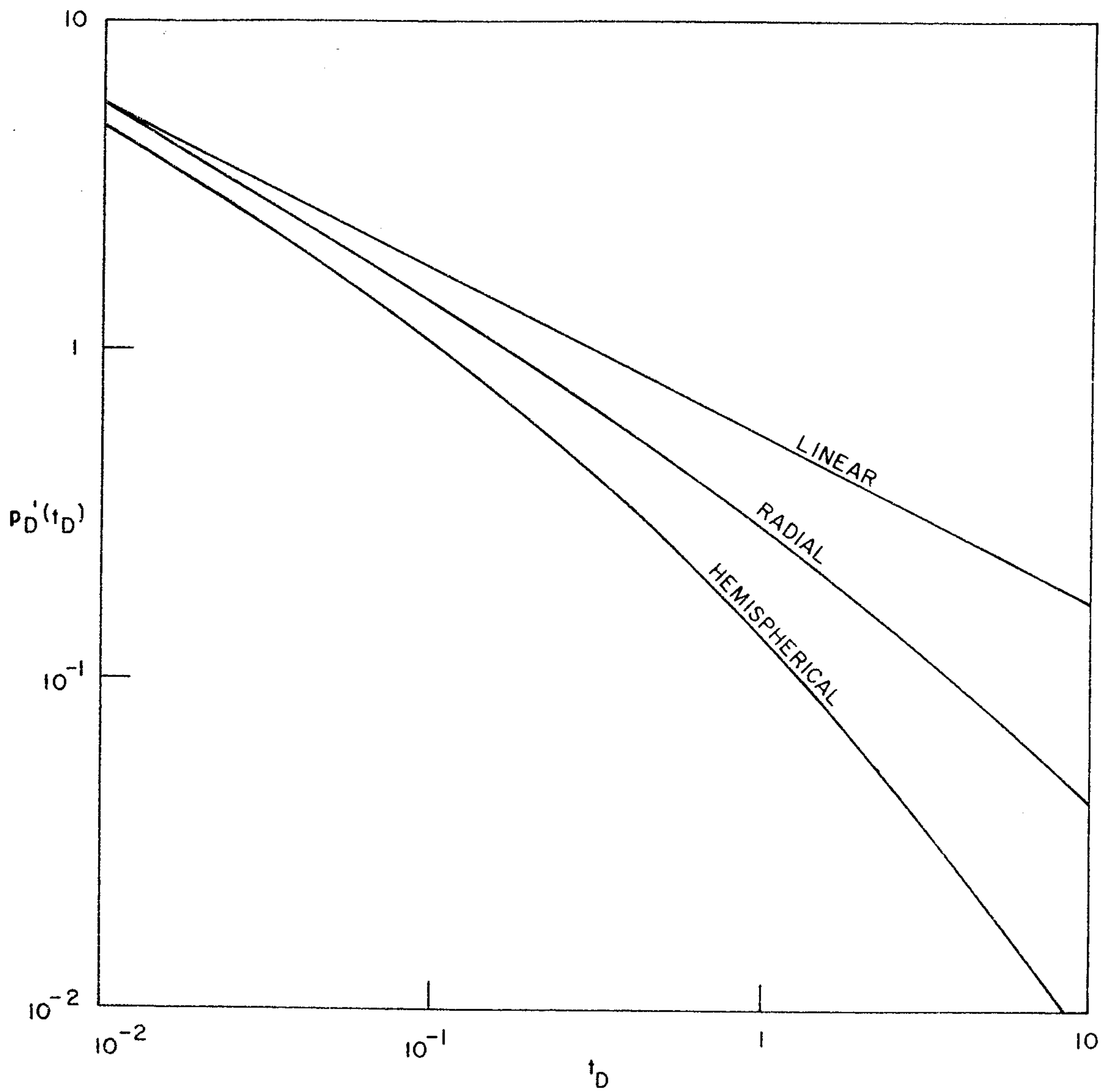

FIG. 20 – $p_D'(t_D)$ VS. t_D FOR VARIOUS AQUIFER GEOMETRIES (INFINITE CASE)

THE CARTER-TRACY INFLUX EQUATION

The gas material balance equation for a water-drive gas reservoir can be written as:

$$W_{en} = G_{pn}B_{gn} - G(B_{gn} - B_{gi}) + B_w W_{pn} \tag{A-10}$$

or

$$W_{en} = B_{gn}(G_{pn} - G) + GB_{gi} + B_w W_{pn} \tag{A-11}$$

since

$$B_{gn} = \frac{Tp_{sc}Z_n}{5.615\ T_{sc}p_n}\ , \tag{A-12}$$

and

$$K = \frac{Tp_{sc}}{5.615\ T_{sc}}\ , \tag{A-13}$$

Eq. A-11 can be written as

$$W_{en} = K\frac{Z_n}{p_n}(G_{pn} - G) + GK\frac{Z_i}{p_i} + B_w W_{pn} \tag{A-14}$$

The Carter-Tracy water influx expression is:

$$W_{en} = W_{e(n-1)} + (t_{Dn} - t_{D(n-1)})\left\{\frac{B\Delta p_n - W_{e(n-1)}p'_D(t_{Dn})}{p_D(t_{Dn}) - t_{D(n-1)}p'_D(t_{Dn})}\right\} \tag{A-15}$$

since

$$\Delta p_n = p_i - p_n \tag{A-16}$$

and

$$C_n = \frac{(t_{Dn} - t_{D(n-1)})}{p_D(t_{Dn}) - t_{D(n-1)}p'_D(t_{Dn})} \tag{A-17}$$

Eq. A-15 can be written as

$$W_{en} = W_{e(n-1)} + C_n[B(p_i - p_n) - W_{e(n-1)}p'_D(t_{Dn})] \tag{A-18}$$

On rearranging

$$W_{en} = C_nBp_i - C_nBp_n + W_{e(n-1)}[1 - C_np'_D(t_{Dn})] \tag{A-19}$$

Combining Eqs. A-14 and A-19:

$$K\frac{Z_n}{p_n}(G_{pn} - G) + GK\frac{Z_i}{p_i} + B_wW_{pn} =$$

$$C_nBp_i - C_nBp_n + W_{e(n-1)}[1 - C_np'_D(t_{Dn})] \tag{A-20}$$

or

$$C_nBp_n = C_nBp_i + W_{e(n-1)}[1 - C_np'_D(t_{Dn})] - GK\frac{Z_i}{p_i}$$

$$- B_wW_{pn} - K\frac{Z_n}{p_n}(G_{pn} - G) \tag{A-21}$$

Multiplying both sides by p_n and rearranging:

$$C_nBp_n^2 + p_n\{GK\frac{Z_i}{p_i} + B_wW_{pn} - W_{e(n-1)}[1-C_np'_D(t_{Dn})]$$

$$- C_nBp_i\} + KZ_n(G_{pn}-G) = 0 \qquad \text{(A-22)}$$

Since

$$E_n = \{GK\frac{Z_i}{p_i} + B_wW_{pn} - W_{e(n-1)}[1-C_np'_D(t_{Dn})] - C_nBp_i\}$$

$$\text{(A-23)}$$

Eq. A-22 can be written as

$$C_nBp_n^{\ 2} + p_nE_n + KZ_n(G_{pn}-G) = 0 \qquad \text{(A-24)}$$

or

$$p_n^{\ 2} + \frac{E_n}{C_nB}\,p_n - \frac{(G-G_{pn})KZ_n}{C_nB} = 0 \qquad \text{(A-25)}$$

and the solution is:

$$p_n = \{-\frac{E_n}{2C_nB} + \frac{1}{2}\sqrt{(\frac{E_n}{C_nB})^2 + \frac{4(G-G_{pn})KZ_n}{C_nB}}\} \qquad \text{(A-26)}$$

The plus sign is chosen since p_n must be positive. Eq. A-26 is rewritten to obtain the Eq. 11 in the main text as:

$$p_n = \frac{1}{2C_nB}\{-E_n + \sqrt{E_n^2 + 4(G-G_{pn})KZ_nC_nB}\} \qquad \text{(A-27)}$$

THE END POINT EQUATION

The end point or abandonment condition is established by a materials balance as:

[Cumulative gas produced at abandonment] =

[Initial gas in place] - [Final gas in place]

(A-28)

or

$$G_{pm} = G - [\text{residual gas in water swept region} + \text{gas in unswept region}] \qquad \text{(A-29)}$$

or

$$G_{pm} = G - \frac{(\text{original PV})(E_p)(S_{gr})}{B_{gm}} - \frac{(\text{original PV})(1-E_p)(1-S_w)}{B_{gm}} \qquad \text{(A-30)}$$

Since

$$G = \frac{(\text{original PV})(1-S_w)}{B_{gi}} \qquad \text{(A-31)}$$

Eq. A-30 becomes

$$G_{pm} = G - \frac{GB_{gi}E_pS_{gr}}{(1-S_w)B_{gm}} - \frac{GB_{gi}(1-E_p)}{B_{gm}} \qquad \text{(A-32)}$$

Substituting $\frac{B_{gi}}{B_{gm}} = \frac{p_m}{p_i}\frac{Z_i}{Z_m}$ in Eq. A-32:

$$G_{pm} = G - GE_p\{\frac{p_mZ_i}{p_iZ_m}(\frac{S_{gr}}{1-S_w} + \frac{1-E_p}{E_p})\} \qquad \text{(A-33)}$$

On rearranging, we get Eq. 17 in the main text as:

$$G_{pm} = G[1-E_p\{\frac{S_{gr}}{S_g} + \frac{(1-E_p)}{E_p}\}\frac{p_mZ_i}{p_iZ_m}] \qquad \text{(A-34)}$$

THE MODIFICATION OF HURST'S SOLUTION

Hurst's solution for water-drive reservoirs (Eq. 43 in Ref. 5) can be modified to apply to a water-drive gas reservoir as:

$$2\pi\phi c_w h r_w^2 \int_0^{t_D} \frac{d\Delta p}{d\tau} Q_D(t_D-\tau)d\tau = G_p B_{gi} - GB_{gi}\bar{c}_g \Delta p \qquad \text{(A-35)}$$

or

$$B\int_0^{t_D} \frac{d\Delta p}{d\tau} Q_D(t_D-\tau)d\tau = \frac{\phi\mu_w c_w r_w^2 B_{gi} q_g t_D}{k} - GB_{gi}\bar{c}_g \Delta p \qquad \text{(A-36)}$$

where $\bar{c}_g$ is an average gas compressibility. This is exactly analogous to Hurst's case for water influx in an undersaturated oil reservoir. The answer in terms of his results is:

$$\Delta p = \frac{\phi\mu_w c_w r_w^2 q_g}{k\ G\ \bar{c}_g} N(\sigma, t_D) \qquad \text{(A-37)}$$

where

$$\sigma = \frac{2\pi\phi c_w h r_w^2}{GB_{gi}\bar{c}_g} \qquad \text{(A-38)}$$

$$N(\sigma,t_D) = \frac{4\sigma}{\pi^2}\int_0^\infty \frac{(1-e^{-u^2t_D})du}{u^3\{[\sigma J_1(u)-uJ_0(u)]^2+[\sigma Y_1(u)-uY_0(u)]^2\}} \tag{A-39}$$

where $J_0(u)$ and $J_1(u)$ are the Bessel functions of the first kind of zero and unit orders. $Y_0(u)$ and $Y_1(u)$ are the Bessel functions of the second kind of the respective orders. $\sigma N(\sigma,t_d)$ is plotted in Fig. 2 of Ref. 5.

Hurst's modified method assumes a constant rate of production. Further, it is necessary to assume that some average gas compressibility, $\bar{c}_g$, applies for the gas reservoir. This method is rather restrictive for gas reservoir calculations.

EXAMPLE PROBLEM--INVESTIGATION OF FACTORS AFFECTING THE VALUES OF SYSTEM CONSTANTS IN PERFORMANCE MATCHING OF A WATER-DRIVE GAS RESERVOIR

To investigate potential variations in the values of the system constants (initial gas in place, G, and water influx constant, B) obtained by using various forms of the material balance equation (Eqs. 19 through 26 in the main text), a reservoir of 5000 acres in area surrounded by an infinitely-large aquifer was selected. Initial reservoir pressure was 7000 psia. The field production rate was 50 MMscf/D; residual gas saturation was 35 per cent of pore volume; and volumetric invasion efficiency was 85 per cent of the bulk volume. Detailed reservoir and aquifer conditions selected are presented in Tables I and II. The permeability of the aquifer was considered to be 5 or 20 md. This particular problem was selected because it would give a significant pressure drop.

To generate the performance history, several computations were required. A six-month time interval was used and the dimensionless time, t_D, was calculated from Eq. 7, employing the conditions selected for the problem. The classic superposition method was used and the pressure values were chosen as specified by van Everdingen et al.[23]

The classic transient flow superposition equation, Eq. 10 in the main text, can be rewritten as:

$$W_{en} = B \sum_{j=0}^{n-1} \Delta p_j Q_D[t_{D(n-j)}] \qquad \text{(A-40)}$$

where

$$Q_D[t_{D(n-j)}] = \text{the value of dimensionless cumulative influx at dimensionless time, } t_{D(n-j)} \qquad \text{(A-41)}$$

According to van Everdingen *et al.*:[23]

$$\Delta p_j = \frac{p_{j-1} - p_{j+1}}{2} , \qquad j = \geq 1 \qquad \text{(A-42)}$$

and

$$\Delta p_0 = \frac{p_0 - p_1}{2} \qquad \text{(A-43)}$$

where

$$p_0 = p_i = \text{initial reservoir pressure} \qquad \text{(A-44)}$$

The above method was applied in the study of two cases. In one case the aquifer permeability was 5 md and in the other case it was 20 md. A constant gas production rate of 50 MMscf/D was assumed for both cases. Pressures at equal increments of time (at end of six-month period) were computed as outlined above. A complete set of performance history was generated using known values of the system constants. The performance history thus obtained was assumed to be the actual performance history of this particular case. A digital computer was employed for all calculations. Graphical results for the two cases (5 md and 20 md) have been shown in Fig. 21 by solid lines.

In order to investigate the effect of various equivalent forms of the material balance equation on the estimate of the system constants, G and B, the above selected problem was used for performance matching. Various portions (25 per cent, 50 per cent and 100 per cent) of the pre-computed history were considered. These various portions of the performance history represent different numbers of data points depending on the length of the history. Note that these data points are the time points at which pressures are computed. For example: in the 5-md case the number of data points (at end of six-month period) were 15, 31 and 62, corresponding to the performance history of about 25 per cent, 50 per cent and 100 per cent. These data points are also shown in Tables VII and VIII for the two cases. Least-mean-square techniques were used to determine the system constants. Only two constants, G and B, were considered. The dimensionless time, t_D, was kept constant for each case. Detailed results are shown in Table VII and VIII. Clearly, it can be seen that values of the system constants vary with the

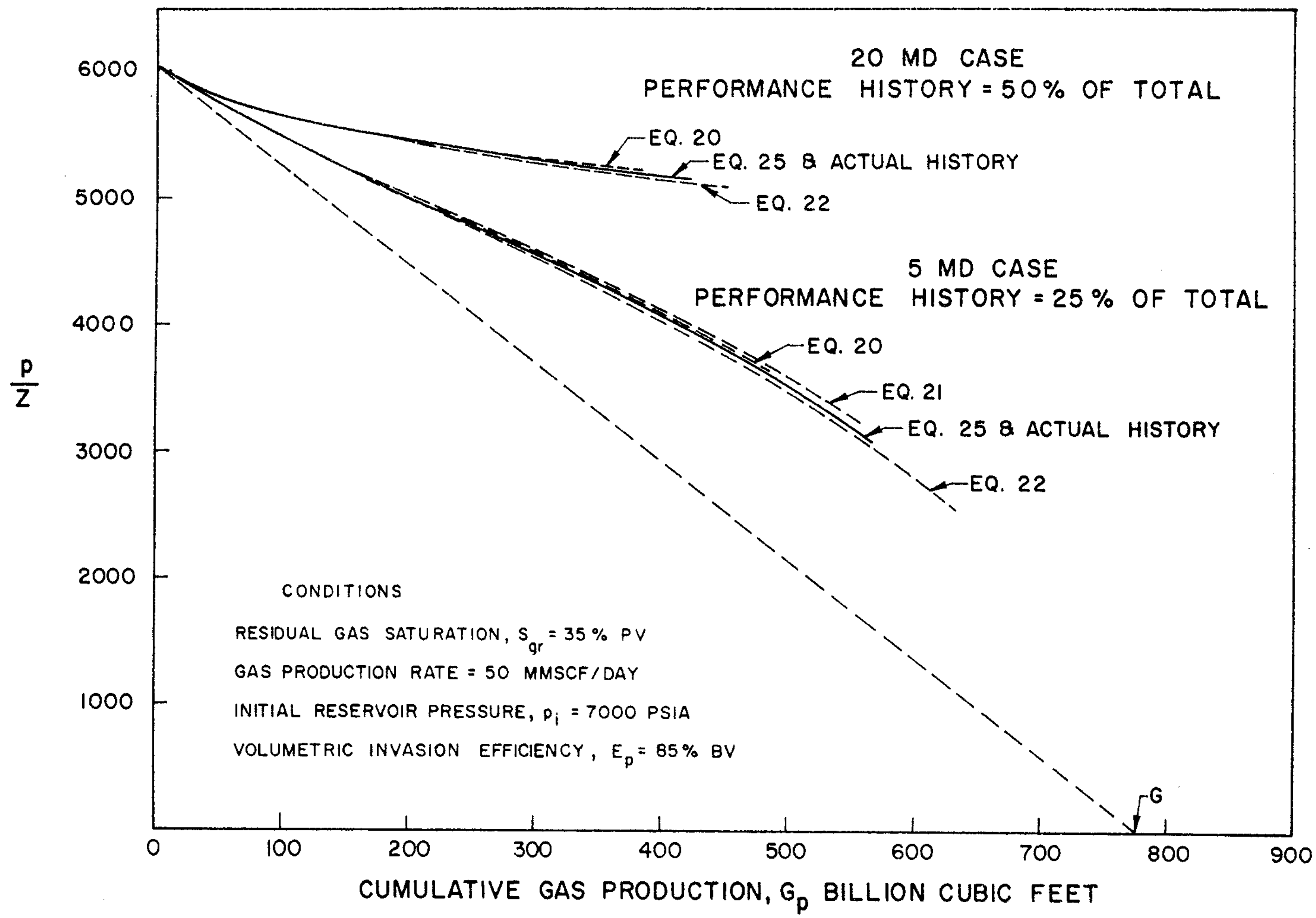

FIG. 21 - COMPUTED $\frac{p}{Z}$ VS. G_p FOR VARIOUS SETS OF SYSTEM CONSTANTS AND AQUIFER PERMEABILITIES

TABLE VII

SUMMARY OF SYSTEM CONSTANTS--5 MD CASE

Initial Gas in place,	G	= 779.98 billion cu ft
Water influx constant,	B	= 7273.59 bbl/psi
Dimensionless time,	t_D	= 0.2536
Total performance history		= 31 years

Estimated System Constants, when Number of Data Points and Percentage of Past History in Relation to the Total History are

Equation	15 (25%)		31 (50%)		62 (100%)	
	G	B	G	B	G	B
19	798.16	6852.60	785.39	7173.73	780.82	7258.47
20	716.71	8385.61	749.78	7709.93	779.67	7271.86
21	784.39	7572.70	788.67	7264.53	790.25	7159.11
22	818.51	6409.15	810.87	6680.71	807.00	6797.77
23	781.89	7236.97	780.83	7256.87	778.95	7291.05
24	765.48	7572.16	777.11	7317.34	778.98	7290.43
25	780.93	7259.00	780.54	7261.99	779.02	7289.88
26	773.32	7416.37	778.98	7287.54	779.02	7289.95

TABLE VIII

SUMMARY OF SYSTEM CONSTANTS--20 MD CASE

Initial Gas in place, G = 779.98 billion cu ft
Water influx constant, B = 7273.59 bbl/psi
Dimensionless time t_D = 1.0144
Total performance history = 23 years

Equation	Estimated System Constants, when Number of Data Points and Percentage of Past History in Relation with the Total History are: 23 (50%) G	23 (50%) B	46 (100%) G	46 (100%) B
19	791.31	7203.47	782.46	7263.62
20	723.64	7491.05	757.98	7327.77
21	787.59	7357.11	790.66	7275.34
22	822.64	7001.89	805.63	7161.44
23	781.13	7266.70	780.14	7272.97
24	765.56	7327.37	777.57	7280.74
25	780.22	7272.14	780.09	7273.19
26	775.99	7293.34	779.57	7274.82

form of the material balance equation, aquifer permeability and percentage of past history used. Although not shown, the results will also vary with the geometry of the aquifer, the size of time increments and the manner in which the pressures are selected for matching.

The minimum deviation between calculated and observed pressures should serve as the criterion for selection of the best form of the material balance equation. The per cent deviation was computed as:

$$\text{per cent deviation} = \Sigma \frac{|p_{observed} - p_{calculated}|}{p_{observed}} \times 100 \qquad \text{(A-45)}$$

Table IX presents per cent absolute deviation as a function of form of the material balance equation for different amounts of past history used. Results are shown for two values of aquifer permeability (5 md and 20 md). Although the per cent deviations are quite small, inspection of Table IX reveals interesting trends. Clearly, per cent deviation should decrease with increasing amounts of past history for each form of the material balance equation used. It also shows that some forms of the equation are more sensitive to the amount of performance history used, as far as the estimation of the system constants is concerned. Note that computed absolute deviation, for the

TABLE IX

RESULTS OF ABSOLUTE DEVIATION BETWEEN CALCULATED AND OBSERVED RESERVOIR PRESSURES

	Per Cent Absolute Deviation as a Function of Per Cent Past History Used for Aquifer Permeability, k_w, of				
	5 md			20 md	
Equation	25%	50%	100%	50%	100%
19	0.3307	0.0322	0.0044	0.0655	0.0069
20	0.2966	0.2407	0.0223	0.1379	0.0060
21	1.1399	0.4437	0.2006	0.2287	0.0877
22	0.6075	0.2199	0.1286	0.2581	0.0795
23	0.0140	0.0071	0.0048	0.0063	0.0004
24	0.1627	0.0192	0.0046	0.0392	0.0058
25	0.0067	0.0062	0.0044	0.0013	0.0003
26	0.0885	0.0111	0.0045	0.0155	0.0010

5-md case shown in Table IX, ranges from 1.140 to 0.201 per cent for the straight line equation (Eq. 21) whereas it ranges only from 0.007 to 0.004 per cent for Eq. 25. These results give a definite indication that Eq. 25 should be a better form of the equation to be used for matching procedures.

Fig. 21 presents p/Z vs cumulative gas produced for various sets of system constants and aquifer permeabilities. These sets of system constants were obtained from various forms of the material balance equation employed for matching. The conditions selected are the same as presented earlier. Results indicate that some forms (Eqs. 20 and 22) of the material balance equation give unreliable estimates of ultimate recovery. Eq. 20 gave a low estimate, whereas Eq. 22 gave high estimate for the system constants and ultimate gas recovery. Nevertheless, it can be seen that there is excellent agreement between actual performance and calculated performance in almost all the cases. Fig. 21 indicates that estimated ultimate recovery and future reservoir performance from Eq. 21 may be reasonably good even using poor system constants. However, it does not appear desirable to obtain good answers with incorrect data.

The above comments deserve further qualification. Reasonably good estimates of the system constants are usually desirable in the early life of a field, when past history is limited. On these estimates will depend development, exploitation and planning of a particular field to optimize gas reserves. It appears that this can be accomplished partly by using suitable forms (Eqs. 23 and 25) of the material balance equation. In the above case, Eq. 25 gave the best results. It should also be evident that significant variations in results may be due to the computing techniques used in matching past history, even though equations which are apparently similar are used.