SPE 110821

Inflow Performance Relationship (*IPR*) For Solution Gas-Drive Reservoirs — Analytical Considerations

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This paper was prepared for presentation at the 2007 SPE Annual Technical Conference and Exhibition held in Anaheim, California, U.S.A., 11–14 November 2007.

Abstract

This work provides the analytical development of "Vogel"-type Inflow Performance Relation (or *IPR*) correlations for solution gas-drive reservoir systems using characteristic flow behavior.

Specifically, we provide the following results:

- An analytical form of the *quadratic* (Vogel) *IPR* correlation:

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\bar{p}}\right] - (1-\nu)\left[\frac{p_{wf}}{\bar{p}}\right]^2$$

  Where the ν-parameter is defined for the solution gas-drive reservoir case using the oil mobility function (*i.e.*, $[k_o/(\mu_o B_o)]$) — this definition is given by:

$$\nu = \frac{2\,[k_o/(\mu_o B_o)]_{\bar{p}=0}}{[k_o/(\mu_o B_o)]_{\bar{p}} + [k_o/(\mu_o B_o)]_{\bar{p}=0}} \text{ or } \frac{1}{(1+\tau\,\bar{p})}$$

- The analytical form for a *cubic IPR* correlation:

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\bar{p}}\right] - \nu\tau\,\bar{p}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \nu\beta\,\bar{p}^2\left[\frac{p_{wf}^3}{\bar{p}^3}\right]$$

  Where the ν-parameter is given by:

$$\nu = \frac{1}{(1+\tau\,\bar{p}+\beta\,\bar{p}^2)}$$

- The analytical form for a *quartic IPR* correlation:

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\bar{p}}\right] - \nu\tau\,\bar{p}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \nu\beta\,\bar{p}^2\left[\frac{p_{wf}^3}{\bar{p}^3}\right] - \nu\eta\,\bar{p}^3\left[\frac{p_{wf}^4}{\bar{p}^4}\right]$$

  Where the ν-parameter is given by:

$$\nu = \frac{1}{(1+\tau\,\bar{p}+\beta\,\bar{p}^2+\eta\,\bar{p}^3)}$$

The practical value of this work is that we have proven that an *IPR* can be written for a given solution gas-drive reservoir system directly from rock-fluid properties and fluid properties.

The "theoretical" value of this work is that we provide a "characteristic" formulation of the oil mobility profile $[k_o/(\mu_o B_o)]$, which is given as:

$$\left[1 - \frac{[k_o/(\mu_o B_o)]_{\bar{p}} - [k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i} - [k_o/(\mu_o B_o)]_{p_{abn}}}\right] =$$

$$1 - \zeta\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right] + (1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^2 - 2(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$

$$(\zeta \le 1)$$

This proposed "characteristic" mobility model is validated against numerical simulation results from the literature and from work performed as part of this study. Note that the characteristic mobility is only a function of the characteristic parameter (ζ), the initial, abandonment and average reservoir pressures (p_i, p_{abn}, and $\bar{p}$), and the oil-phase mobility evaluated at the initial and the abandonment reservoir pressure $[k_o/(\mu_o B_o)]_{p_i}$ and $[k_o/(\mu_o B_o)]_{p_{abn}}$.

Introduction

In 1968 Vogel [Vogel (1968)] established an empirical relationship for flowrate prediction of a solution gas-drive reservoir in terms of the wellbore pressure based on reservoir simulation results. This may seem trivial because we can write analytical results (*i.e.*, *IPR* formulations) for the slightly compressible liquid case as well as the dry gas reservoir case. However, the development of an analytical result for the solution gas-drive case requires the use of the oil-phase pseudopressure which is written as follows:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp \quad \text{............................ (1)}$$

A variation of Eq. 1 was presented by Evinger and Muskat [Evinger and Muskat (1942)] for steady-state flow. The dilemma then, as now, is the issue of the effective (or relative permeability) term — the dependence of effective/relative permeability on saturation requires that the saturation distribution be known — which (of course) it is not.

The logical step forward (at least for Vogel) was to correlate the flowrate-pressure behavior in much the same fashion as one would for the single-phase liquid or gas case — using a pseudosteady-state flow model. For a solution gas-drive reservoir the pseudosteady-state flow model for the oil phase is written as: [Camacho (1987), Camacho and Raghavan (1989, 1991)]

$$q_o = \frac{1}{b_{pss}} [\, p_{po}(\bar{p}) - p_{po}(p_{wf})\,] \qquad (2)$$

Eq. 2 is not particularly useful as it requires the computation of Eq. 1 — and, as noted, Eq.1 requires that the oil mobility function $[k_o/(\mu_o B_o)]$ be known continuously as a function of pressure and saturation. Hence, Vogel proceeded to develop an empirical "pseudosteady-state" flow equation in the form of a scaled flowrate and pressure function based on an extensive sequence of reservoir simulation cases. The general form of the Vogel "*IPR* correlation" is given as:

$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{\bar{p}}\right] - (1-v)\left[\frac{p_{wf}}{\bar{p}}\right]^2 \qquad (3)$$

Where Vogel developed a reference curve using Eq. 3 and selected v=0.2 as the "reference" value (see **Fig. 1**).

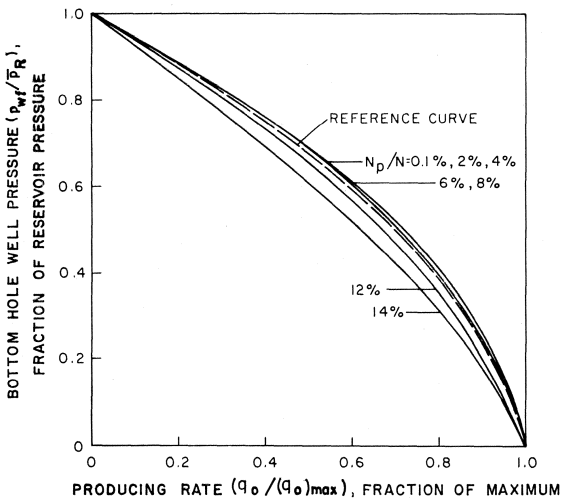

Figure 1 — *IPR* behavior for solution-gas drive systems at various stages of depletion — the "reference curve" is the correlation presented by Vogel [Vogel (1968)].

In 1973 Fetkovich [Fetkovich (1973)] derived a "pressure-squared" deliverability relation using pseudosteady-state theory and a presumed linear relationship for the liquid (oil) mobility function (*i.e.*, $[k_o/(\mu_o B_o)]$). The Fetkovich "deliverability" relation is given as:

$$\frac{q_o}{q_{o,\max}} = \left[1 - \left[\frac{p_{wf}}{\bar{p}}\right]^2\right]^n \qquad (4)$$

Fetkovich proposed Eq. 4 as a "simpler," yet theoretically consistent alternative to the Vogel *IPR* formulation (Eq. 3). Fetkovich compared Eq. 4 to Eq. 3 for practical applications and produced **Fig. 2** as a rationale for his preference of Eq. 4.

We discuss the Vogel and Fetkovich proposals in the context of what an Inflow Performance Relation (or *IPR*) represents — a *correlation* of flowrate and pressure performance. Attempts to derive or theoretically validate these relations [Camacho (1987), Camacho and Raghavan (1991), Wiggins *et al* (1996)] all resort to some type of an approximation or condition under which an *IPR* could be considered "applicable."

The generic goal of our present work is to provide a theoretical basis for the concept of an *IPR* — but to do so in a fashion that establishes what an *IPR* is (*i.e.*, a correlation) and what an *IPR* is not (*i.e.*, a rigorous flow equation). Ultimately, we would like to provide a consistent understanding of why the Vogel (quadratic) *IPR* form functions so effectively in practice. As part of that effort we provide a quasi-analytical derivation of the Vogel *IPR* — specifically, we provide an approximate result in the form of the traditional Vogel (quadratic) *IPR* form (*i.e.*, Eq. 3) as well as an analytical basis for the v-parameter (**Appendix A**).

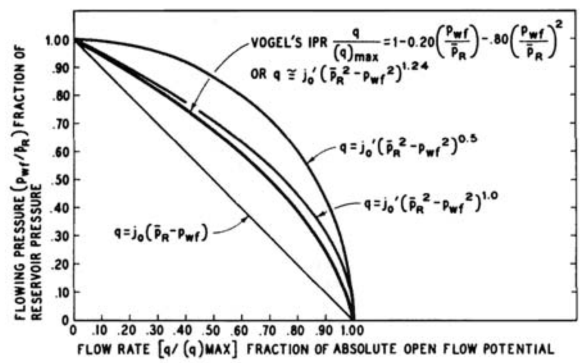

Figure 2 — Inflow performance relations for various flow equations [Fetkovich (1973)].

The basis for the Vogel quadratic *IPR* form is that assumption that the mobility profile is linear (obviously for $p<p_b$), as given below:

$$[k_o/(\mu_o B_o)]_{\bar{p}} = f(\bar{p}) = a + 2b\,\bar{p} \qquad (5)$$

Where a and b are *constants* established from the presumed behavior of the mobility profile. The first literature citation of Eq. 5 is by Fetkovich [Fetkovich (1973)], where Fetkovich used this formulation to develop his "deliverability" equations for solution gas-drive systems. For a graphical representation of Eq. 5, we cite **Fig. 3**, originally proposed by Fetkovich.

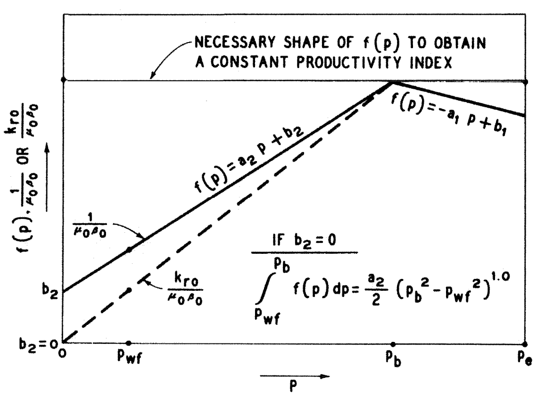

Figure 3 — Schematic mobility-pressure behavior for solution-gas drive reservoirs [Fetkovich (1973)].

As we consider the next steps in our *IPR* validation, we return to the salient work by Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)] — where they utilized numerical simulation to characterize generalized flow behavior in solution gas-drive reservoir systems.

Perhaps the most important contribution made by Camacho and Raghavan in their work on "well deliverability" was their presentation of the behavior of the oil mobility profile as a function of pressure. In particular, Camacho and Raghavan had the insight to "normalize" the mobility and pressure data to their respective initial values. This provides a unique signature of the behavior of solution gas-drive systems as shown in **Fig. 4**.

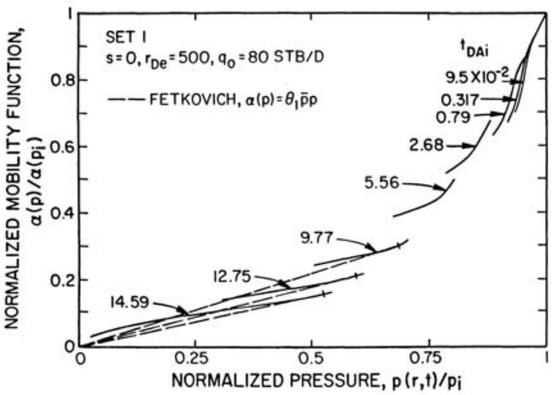

Figure 4 — Mobility performance for a solution gas-drive reservoir system [Camacho (1987), Camacho and Raghavan (1989, 1991)].

The most striking aspect of **Fig. 4** is the character of the mobility profile — in particular, the inapplicability of the "Fetkovich" linear mobility profile (*i.e.*, Eq. 5) (note the linear trends projected on to the data at late times (*i.e.*, low pressures)). In fact, **Fig. 4** *confirms that the "linear" mobility function does not exist at early times/high pressures* (even if the reservoir is in boundary-dominated flow — for reference, the start of boundary-dominated is approximately t_{DAi}=0.1).

We use the "normalized" format given by **Fig. 4** to resolve the character of the mobility function ($[k_o/(\mu_o B_o)]$) so that we can use extend the Vogel concept to include more general (and more accurate) representations of the mobility function.

Characteristic Behavior of Solution Gas-Drive Reservoir Systems

In this section we provide validation of the characteristic behavior of solution gas-drive reservoir systems using reservoir simulation results at reservoir and average reservoir pressures. We first provide a general correlating relation for the mobility function — which is a polynomial expansion (analogous to a geometric series) based on a single parameter (ζ). The correlation is "normalized" to the initial and abandonment pressure (p_i and p_{abn}) and is written as:

$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\bar{p}}-[k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i}-[k_o/(\mu_o B_o)]_{p_{abn}}}\right]=$$

$$1-\zeta\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^2-2(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$

$$(\zeta \le 1) \qquad (6)$$

The basis for Eq. 6 is our "recast" of **Fig. 4**, given now in terms of (1 - $[(k_o/(\mu_o B_o))_{avg}$ - $(k_o/(\mu_o B_o))_{abn}]$ / $[(k_o/(\mu_o B_o))_i$ - $(k_o/(\mu_o B_o))_{abn}]$) — which we will call the "characteristic mobility function." In **Fig. 5** we plot the characteristic mobility function versus $(p(r,t)-p_{abn})/(p_i-p_{abn})$ using the data of Camacho and Raghavan. The next step in our validation process is to reproduce the trends shown in **Fig. 5** *using the same simulation input data as Camacho and Raghavan* [Camacho (1987), Camacho and Raghavan (1989, 1991)]. Our reproduction of the "characteristic mobility function" is shown in **Fig. 6**.

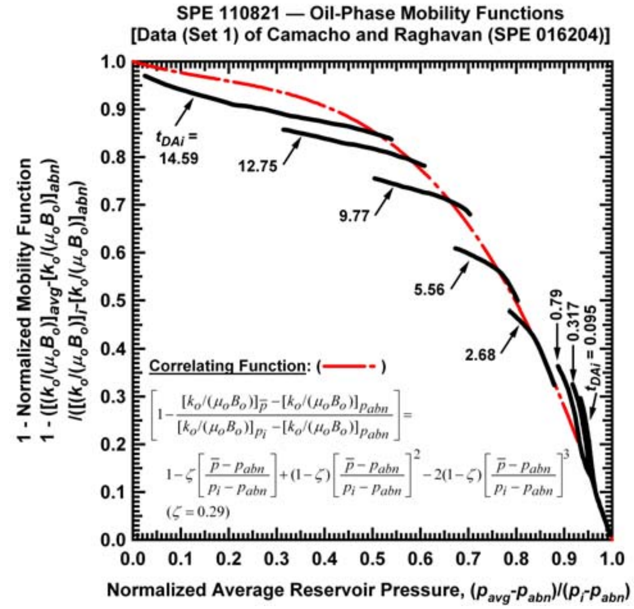

Figure 5 — Mobility performance for a solution gas-drive reservoir system [Camacho (1987), Camacho and Raghavan (1989, 1991)] — recast in terms of 1 minus the normalized mobility function.

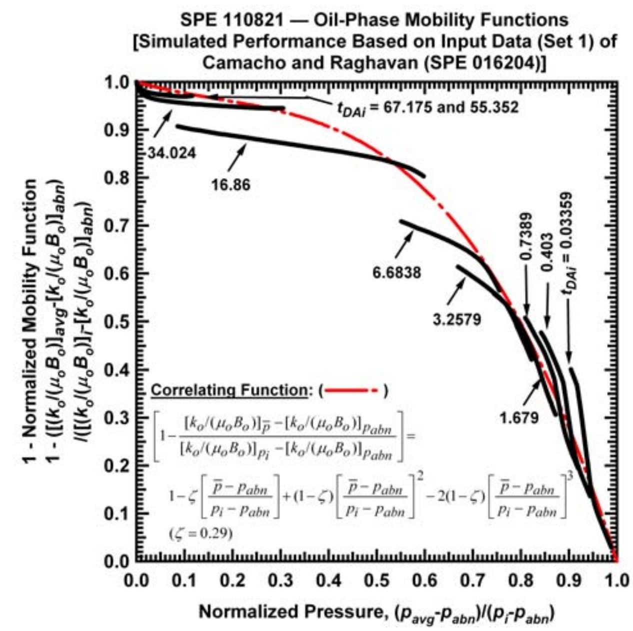

Figure 6 — Mobility performance for a solution gas-drive reservoir system — calibration of reservoir model using input data (set 1) of Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)].

These comparisons are a necessary component of our "calibration" for the *IPR* correlations — if we can uniquely characterize the mobility performance then we can develop a quasi-analytical basis for creating rigorous *IPR* functions. In some ways our logic is akin to that of Wiggins *et al* [Wiggins *et al* (1996)] where their approach was to develop empirical, polynomial expansions of the mobility function.

Our study differs in that our goal (like Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)]) is to identify the "characteristic" mobility behavior for the performance of solution gas-drive reservoirs. Where such behavior will be uniquely (and universally) described by a "characteristic" function. Thus, Eq. 6 evolved from investigations at a "characteristic"-level (*i.e.*, distillation of the "characteristic" mobility behavior into simple, universal relations).

Our next step is to verify that this "characteristic" concept can be extended to the average reservoir pressure condition (*i.e.*, to prove that the characteristic mobility function is also valid for the average reservoir pressure condition). For this investigation we propose a characteristic mobility function in terms of the average reservoir pressure $(\overline{p})$ and the abandonment reservoir pressure (p_{abn}) — where this relation is written as:

$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\overline{p}}-[k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i}-[k_o/(\mu_o B_o)]_{p_{abn}}}\right]=$$

$$1-\zeta\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^2-2(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$

$(\zeta \le 1)$

.... (7)

As Eq. 7 is proposed, we perform a sequence of simulation cases generated using constant rate, constant pressure, and variable-rate conditions. The results of the variable-rate simulation case are formulated in the "characteristic mobility form" (in $(\overline{p})$) and presented in **Fig. 7.**

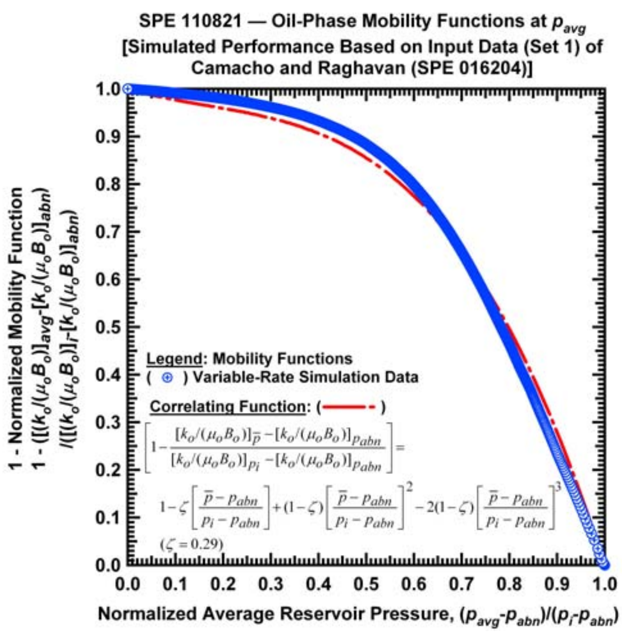

Figure 7 — Mobility performance for a solution gas-drive reservoir system — mobility evaluated at average reservoir pressure. Input data (set 1) of Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)].

Based on the results shown in Fig. 7, we believe that we have established a theoretically consistent characteristic model for mobility (*i.e.*, Eq. 7), from which we can build a unique (and theoretically consistent) *IPR* correlations for the solution gas-drive case.

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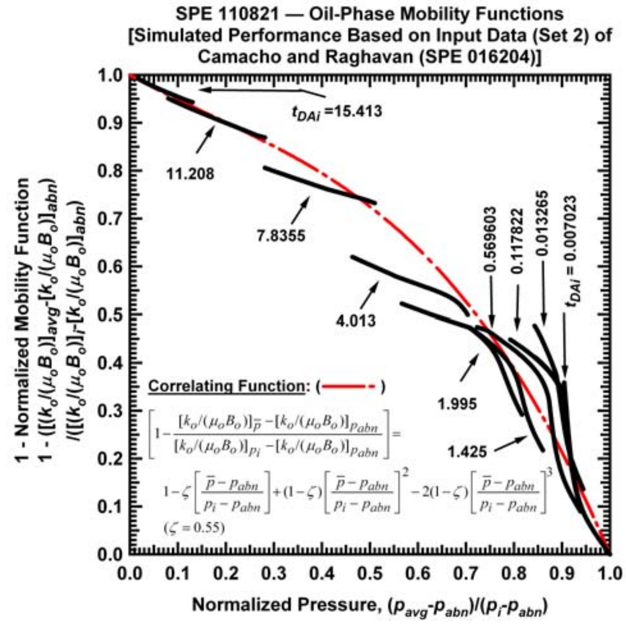

Figure 8 — Mobility performance for a solution gas-drive reservoir system — calibration of reservoir model using input data (set 2) of Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)].

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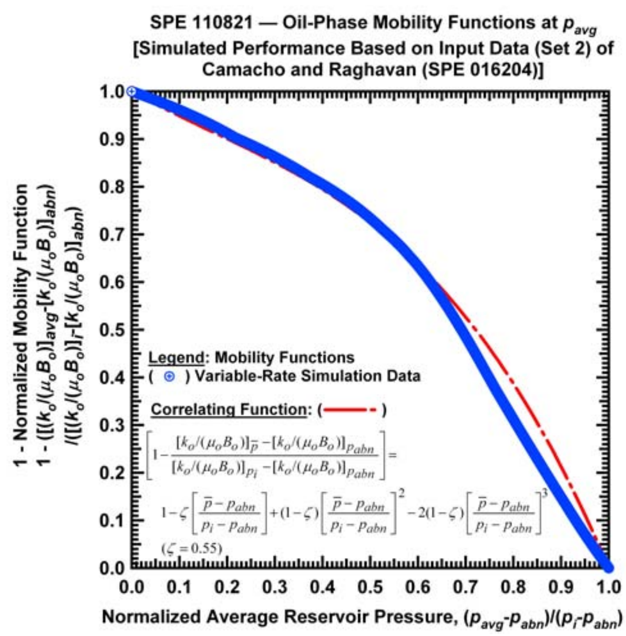

Figure 9 — Mobility performance for a solution gas-drive reservoir system — mobility evaluated at average reservoir pressure. Input data (set 2) of Camacho and Raghavan [Camacho (1987), Camacho and Raghavan (1989, 1991)].

Based on the work described above — we provide a unique correlation of the oil mobility as a characteristic function (*i.e.*, $[k_o/(\mu_o B_o)]_{\overline{p}}$ as described by Eq. 7). Therefore, the parameters required to develop an *IPR* correlation for the solution gas-drive reservoir case are uniquely defined as:

- The characteristic parameter, ζ.
- The initial and abandonment reservoir pressure, p_i, p_{abn}
- The oil mobility at p_i, and p_{abn} $[k_o/(\mu_o B_o)]_{p_i}$ and $[k_o/(\mu_o B_o)]_{abn}$.

IPR Correlations for Solution Gas-Drive Systems

In this section we document the *IPR* models we have developed and we provide orientation as to the basis (*i.e.*, assumptions and limitations) for each *IPR* model.

Vogel (*Quadratic*) *IPR Case*: *Linear* $[k_o/(\mu_o B_o)]_{\bar{p}}$ *profile*

Recalling Eq. 5 (*i.e.*, the specific case of a linear mobility function), we have:

$$[k_o/(\mu_o B_o)]_{\bar{p}} = f(\bar{p}) = a + 2b\,\bar{p} \quad \text{.......... (5)}$$

In **Appendix A** we provide the development of the generic *quadratic* (Vogel) *IPR* case based on the substitution of Eq. 5 into Eq. 1 (the oil-phase pseudopressure function), where that result is then substituted into Eq. 2 (the pseudosteady-state relation for the solution gas-drive reservoir system). After considerable algebraic manipulation, the final result of this process is given as:

$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{\bar{p}}\right] - (1-v)\left[\frac{p_{wf}}{\bar{p}}\right]^2 \quad \text{.......... (8)}$$

Where the v-parameter is defined uniquely for this case in terms of the oil mobility function evaluated at the average reservoir pressure $[k_o/(\mu_o B_o)]_{\bar{p}}$. The specific definition of the v-parameter (for this case) is given by:

$$v = \frac{2[k_o/(\mu_o B_o)]_{\bar{p}=0}}{[k_o/(\mu_o B_o)]_{\bar{p}} + [k_o/(\mu_o B_o)]_{\bar{p}=0}} \quad \text{.......... (9)}$$

Cubic IPR Case: *Quadratic* $[k_o/(\mu_o B_o)]_{\bar{p}}$ *profile*

In **Appendix B** we provide the development of the generic *cubic IPR* formula using as similar procedure as outlined in **Appendix A** for the linear mobility profile case. In this case we employ the *quadratic* $[k_o/(\mu_o B_o)]_{\bar{p}}$ profile to obtain the required result, which is written as:

$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{\bar{p}}\right] - v\tau\,\bar{p}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - v\beta\,\bar{p}^2\left[\frac{p_{wf}^3}{\bar{p}^3}\right] \quad \text{.......... (10)}$$

Where the specific definition of the v-parameter (for this case) is given by:

$$v = \frac{1}{(1 + \tau\,\bar{p} + \beta\,\bar{p}^2)} \quad \text{.......... (11)}$$

Quartic IPR Case: *Cubic* $[k_o/(\mu_o B_o)]_{\bar{p}}$ *profile*

In **Appendix C** we provide the development of the generic *quartic IPR* formula using as similar procedure as outlined in **Appendix A** for the linear mobility profile case. In this case we employ the *quadratic* $[k_o/(\mu_o B_o)]_{\bar{p}}$ profile to obtain the required result, which is written as:

$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{\bar{p}}\right] - v\tau\,\bar{p}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - v\beta\,\bar{p}^2\left[\frac{p_{wf}^3}{\bar{p}^3}\right] - v\eta\,\bar{p}^3\left[\frac{p_{wf}^4}{\bar{p}^4}\right] \quad (12)$$

$$v = \frac{1}{(1 + \tau\,\bar{p} + \beta\,\bar{p}^2 + \eta\,\bar{p}^3)} \quad \text{.......... (13)}$$

We note that Eqs. 10 and 12 (and for that matter, Eq. 8) are all subordinate results based on the concept of the characteristic mobility function discussed earlier, and given in functional form by Eq. 7. We will continue our work process using Eq. 7 and develop a completely generic *IPR* formulation based on the characteristic mobility function.

Summary and Conclusions

Summary: In this work we have provided a comprehensive development and validation of the Inflow Performance Relationship (or *IPR*) concept as proposed by Vogel for the case of a solution gas-drive reservoir.

Our basis for validation in this work is the model of a "characteristic mobility function" which we have developed as a concept-based representation of the mobility-pressure relationship. Specifically, we have shown using the results of numerical simulation that the mobility function at average reservoir pressure, normalized to the initial pressure is a unique function of the average reservoir pressure/initial reservoir pressure.

This "characteristic" behavior can be written as:

$$\left[\frac{[k_o/(\mu_o B_o)]_{\bar{p}} - [k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i} - [k_o/(\mu_o B_o)]_{p_{abn}}}\right] = f\left[\zeta, \frac{\bar{p} - p_{abn}}{p_i - p_{abn}}\right] \quad (\zeta \le 1)$$

We have used this characteristic behavior concept to extend the *IPR* correlation approach to quadratic and cubic mobility profiles (expressed in terms of the ζ-parameter). While we make no claim as to the "analytic" nature of the characteristic mobility behavior, we believe that this behavior does validate the Vogel (quadratic) *IPR* correlation (as an approximation), as well as permit us to extend the *IPR* correlation concept to higher-order formulations.

Put simply, the characteristic mobility concept allows us to develop "near-analytic" relations for the pseudosteady-state flow behavior of solution gas-drive reservoir systems. While not an objective of this work, the proposed developments could have value in developing rate-time formulas for the boundary-dominated flow performance of solution gas-drive reservoir systems.

Conclusions:

1. A general form of the Vogel (quadratic) *IPR* correlation can be derived using the assumption of a linear mobility profile (analogous to the derivation of the pressure-squared "deliverability" equation as proposed by Fetkovich [Fetkovich (1973)] for the solution gas-drive reservoir case).
2. The characteristic mobility parameter (ζ) uniquely defines the mobility profile for the performance of a solution gas-drive reservoir.
3. The cubic and quartic *IPR* formulations derived using the quadratic and cubic expansions for oil-phase mobility are

considered unique as these results were derived based on the concept of the characteristic mobility function.

Nomenclature

Variables

- a = Constant established from the presumed behavior of the mobility profile.
- b = Constant established from the presumed behavior of the mobility profile.
- b_{pss} = Pseudosteady-state flow constant.
- B_g = Gas formation volume factor, RB/SCF
- B_o = Oil formation volume factor, RB/STB
- ϕ = Porosity, fraction
- h = Pay thickness, ft
- k = Absolute permeability, md
- k_o = Relative permeability to oil, fraction
- k_{ro} = Effective permeability to oil, md
- $\overline{p}$ = Average reservoir pressure, psia
- p_{abn} = Abandonment pressure, psia
- p_{base} = Base pressure, psia
- p_n = Reference pressure, psia
- p_i = Initial reservoir pressure, psia
- p_{po} = Oil pseudopressure, psia
- p_{wf} = Flowing bottomhole pressure, psia
- q_o = Oil flowrate, STB/D
- $q_{o,\max}$ = Maximum Oil flowrate, STB/D
- R_{so} = Solution gas-oil ratio, SCF/STB
- r_e = Outer reservoir radius, ft
- r_w = Wellbore radius, ft
- s = Skin factor, dimensionless
- S_g = Gas saturation, dimensionless
- S_o = Oil saturation, dimensionless

Greek Symbols

- β = General *IPR* "lump" parameter, dimensionless
- χ = Linear *IPR* "lump" parameter, dimensionless
- η = General *IPR* "lump" parameter, dimensionless
- μ_g = Gas viscosity, cp
- μ_o = Oil viscosity, cp
- ν = General *IPR* "lump" parameter, dimensionless
- τ = General *IPR* "lump" parameter, dimensionless
- ζ = Characteristic mobility parameter, dimensionless

Oil Pseudofunction:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp$$

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Appendix A: Derivation of a General Quadratic Inflow Performance Relationship (*IPR*) for Solution Gas-Drive Reservoirs Using a Linear Model for the Oil Mobility Function (Alternate Approach to Fetkovich)

In this Appendix we show that an inflow performance relationship (*IPR*) can be developed based on the pseudosteady-state flow equation for a single well in a solution gas-drive reservoir (based on the oil-phase pseudopressure formulation) and using an approximate relation for the mobility of the oil phase. Elements of this derivation are taken from Del Castillo [Del Castillo (2003)], where Del Castillo considered the case of gas condensate reservoirs — but used the Vogel-type *IPR* form as a starting point for her work.

The definition of the oil-phase pseudopressure for a single well in a solution gas-drive reservoir is given as:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp \quad \text{.......................} \text{(A-1)}$$

The pseudosteady-state flow equation for the oil-phase in a solution gas-drive reservoir is given by:

$$p_{po}(\overline{p}) = p_{po}(p_{wf}) + q_o b_{pss} \quad \text{...} \text{(A-2)}$$

Where the "pseudosteady-state" constant (b_{pss}) is given by:

$$b_{pss} = 141.2 \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \frac{1}{h}\left[\ln\left[\frac{r_e}{r_w}\right] - \frac{3}{4} + s\right] \quad \text{......................} \text{(A-3)}$$

For the solution gas-drive case, we propose the following model for the oil mobility function, $\left[k_o/(\mu_o B_o)\right]_p$:

$$\left[\frac{k_o}{\mu_o B_o}\right]_{\overline{p}} = f(\overline{p}) = a + 2b\overline{p} \quad \text{..} \text{(A-4)}$$

We note that our proposed model for the oil mobility function given in Eq. A-4 is very similar to the relation proposed by Fetkovich [Fetkovich (1973)] for the case of a solution gas-drive reservoir system. We also note that Fetkovich utilized a "zero intercept" for the development of his oil-phase deliverability equation (*i.e.*, the mobility at zero pressure is zero (see **Fig. A.1**)).

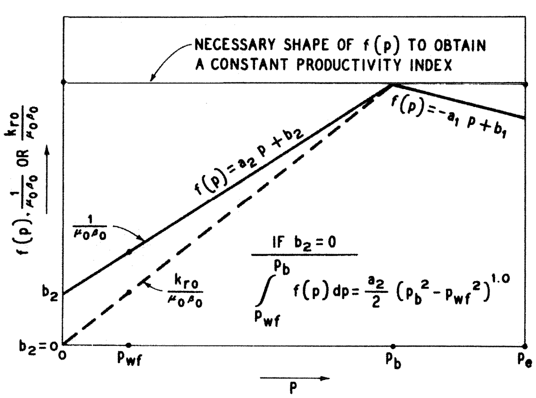

Figure A.1 — Mobility-pressure behavior for a solution gas-drive reservoir [Fetkovich (1973)].

In our proposal (*i.e.*, Eq. A-4), we do not presume a zero intercept of the mobility function — from **Fig. A.1** we conclude that the zero mobility at zero pressure was based on the assumption (by Fetkovich) that at zero pressure the k_{ro} term would be zero (*i.e.*, no oil would flow). Using **Fig. A.1** as a guide, we note that our linear mobility concept (*i.e.*, Eq. A-4) is plausible.

We will first establish the *IPR* formulation for the pseudo-pressure form of the oil flow equation for a solution gas-drive system. Solving Eq. A-2 for the oil rate, q_o, we have:

$$q_o = \frac{1}{b_{pss}}[\, p_{po}(\bar{p}) - p_{po}(p_{wf})] \quad \text{(A-5)}$$

Solving Eq. A-5 for the case of the "maximum oil rate," $q_{o,\max}$, (*i.e.*, p_{wf} =0 (or $p_{po}(p_{wf})$ =0)), we have:

$$q_{o,\max} = \frac{1}{b_{pss}}[\, p_{po}(\bar{p}) - p_{po}(p_{wf}=0)] \quad \text{(A-6)}$$

Dividing Eq. A-5 by Eq. A-6 gives us the "*IPR*" form (*i.e.*, $q_o/q_{o,max}$) — which yields:

$$\frac{q_o}{q_{o,\max}} = \frac{p_{po}(\bar{p}) - p_{po}(p_{wf})}{p_{po}(\bar{p}) - p_{po}(p_{wf}=0)} \quad \text{(A-7)}$$

At this point we will note that *it is not our goal* to proceed with the development of an *IPR* model in terms of the pseudopressure function, $p_{po}(p)$ — rather, our goal is to develop a simplified *IPR* model using Eqs. A-4 and A-7 as base relations. Given that Eq. A-4 is given in terms of pressure (p), we can presume that some type of pressure-squared formulation will result (as was the case in the Fetkovich work [Fetkovich (1973)].

Substituting Eq. A.4 into Eq. A.1, we have:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} (a + 2bp)\, dp \quad \text{(A-8)}$$

Or, completing the integration, we obtain:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \left[(ap + bp^2) - (ap_{base} + bp_{base}^2)\right] \quad \text{(A-8)}$$

Substituting Eq. A.8 into Eq. A.7, gives us:

$$A = (a\bar{p} + b\bar{p}^2)$$

$$B = (ap_{base} + bp_{base}^2)$$

$$C = (ap_{wf} + bp_{wf}^2)$$

$$D = (a(0) + b(0)^2)$$

$$\frac{q_o}{q_{o,\max}} = \frac{[A-B]-[C-B]}{[A-B]-[D-B]} \quad \text{(A-9)}$$

Cancelling like terms, we obtain:

$$\frac{q_o}{q_{o,\max}} = \frac{(a\bar{p} + b\bar{p}^2) - (ap_{wf} + bp_{wf}^2)}{(a\bar{p} + b\bar{p}^2)} \quad \text{(A-10)}$$

Dividing through Eq. A-9 by $(a\bar{p} + b\bar{p}^2)$ gives us the following forms:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{(ap_{wf} + bp_{wf}^2)}{(a\bar{p} + b\bar{p}^2)} \quad \text{(A-11)}$$

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{ap_{wf}}{(a\bar{p} + b\bar{p}^2)} - \frac{bp_{wf}^2}{(a\bar{p} + b\bar{p}^2)} \quad \text{(A-12)}$$

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1+\frac{b}{a}\bar{p})}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{a}{b}\frac{1}{\bar{p}}+1)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] \quad \text{(A-13)}$$

Defining $\tau = b/a$

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1+\tau\,\bar{p})}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{1}{\tau}\frac{1}{\bar{p}}+1)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] \quad \text{(A-14)}$$

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1+\tau\,\bar{p})}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{\tau\,\bar{p}}{(1+\tau\,\bar{p})}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] \quad \text{(A-15)}$$

Defining a "lumped parameter," ν:

$$\nu = \frac{1}{(1+\tau\,\bar{p})} \quad \text{(A-16)}$$

Therefore:

$$(1-\nu) = 1 - \frac{1}{(1+\tau\,\bar{p})} = \frac{(1+\tau\,\bar{p})}{(1+\tau\,\bar{p})} - \frac{1}{(1+\tau\,\bar{p})} = \frac{\tau\,\bar{p}}{(1+\tau\,\bar{p})}$$

Or,

$$(1-\nu) = \frac{1}{(1+\frac{1}{\tau\,\bar{p}})} \quad \text{(A-17)}$$

substituting Eqs. A-16 and A-17 into Eq. A-15, we have:

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\bar{p}}\right] - (1-\nu)\left[\frac{p_{wf}}{\bar{p}}\right]^2 \quad \text{(A-18)}$$

Where we note that Eq. A-18 *has exactly the same form as the empirical result proposed by Vogel* [Vogel (1968)]. We suggest that Eq. A-18 serves as a semi-analytical validation of the Vogel result — and while we recognize that the ν-parameter is not "constant," this parameter can be established directly from the proposed model for mobility (*i.e.*, Eq. A-4).

As the ν-parameter is given as a function of the average reservoir pressure, $\bar{p}$, we recall Eq. A-4 and express this result in terms of $\bar{p}$.

$$\left[\frac{k_o}{\mu_o B_o}\right]_{\bar{p}} = a + 2b\bar{p} \quad \text{(A-19)}$$

At $\bar{p}=0$, Eq. A-19 becomes:

$$\left[\frac{k_o}{\mu_o B_o}\right]_{\bar{p}=0} = a$$

Or,

$$a = \left[\frac{k_o}{\mu_o B_o}\right]_{\bar{p}=0} \quad \text{(A-20)}$$

Dividing through Eq. A-19 by the a-parameter, we define a new parameter, χ:

$$\chi = \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} = 1 + 2\frac{b}{a}\,\overline{p} \qquad \text{(A-21)}$$

Or, using the definition $\tau = b/a$, we have:

$$\chi = 1 + 2\tau\overline{p} \qquad \text{(A-22)}$$

Recalling Eq. A-16 (*i.e.*, the definition of the ν-parameter), we have:

$$\nu = \frac{1}{(1+\tau\,\overline{p})} \qquad \text{(A-16)}$$

Solving the "χ" definition (Eq. A-22) for the $\tau\overline{p}$ term gives us:

$$\tau\overline{p} = \frac{\chi - 1}{2}$$

Therefore, the $(1+\tau\overline{p})$ term is given by $\tau\overline{p}$ term gives us:

$$(1+\tau\,\overline{p}) = \frac{2}{2} + \frac{\chi-1}{2} = \frac{\chi+1}{2}$$

And,

$$\frac{1}{(1+\tau\,\overline{p})} = \frac{2}{\chi+1} \qquad \text{(A-23)}$$

We note that Eq. A-16 (*i.e.*, the definition for the ν-parameter) and Eq. A-23 (an equality based on the χ-parameter) are equivalent — which leads to the following definition:

$$\nu = \frac{2}{\chi+1} \qquad \text{(A-24)}$$

A similar relation can be derived for the (1-ν) group directly from Eq. A-24. This derivation is given by:

$$(1-\nu) = \frac{\chi+1}{\chi+1} - \frac{2}{\chi+1}$$

Or, upon algebraic reduction, we have:

$$(1-\nu) = \frac{\chi-1}{\chi+1} \qquad \text{(A-25)}$$

Substitution of Eqs. A-24 and A-25 into the *IPR* model (Eq. A-18) gives the following result in terms of the χ-parameter:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{2}{\chi+1}\left[\frac{p_{wf}}{\overline{p}}\right] - \frac{\chi-1}{\chi+1}\left[\frac{p_{wf}}{\overline{p}}\right]^2 \qquad \text{(A-26)}$$

We note that Eq. A-26 (*i.e.*, the *IPR* model given in terms of the χ-parameter) is presented for completeness — we continue to advocate the "conventional form" of the *IPR* model (*i.e.*, Eq. A-18, which is given in terms of the ν-parameter).

For *compactness*, we will continue to use the χ-parameter as the preferred variable for expressing the mobility function. Recalling the definition of the χ-parameter (Eq. A-21), we have:

$$\chi = \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} \qquad \text{(A-27)}$$

We state explicitly that the χ-parameter is not constant — however, we propose that concept of using a single parameter to represent a particular segment of performance is well-established. We believe that the modified "Vogel" model (Eq. A-18) is directionally correct and does have theoretical justifications (as shown in this Appendix). But we also recognize that this concept requires further proof — particularly from the standpoint of proving that the χ-parameter can be estimated using conventional PVT and relative permeability data.

In our final effort, we propose to define the ν and (1-ν) terms as functions of the mobility parameters. We achieve these definitions using the results from Eq. A-21 (*i.e.*, the base definition) and Eqs. A-24 and A-25 (the ν and (1-ν) definitions, respectively). Substituting Eq. A-21 into Eq. A-25 gives:

$$(1-\nu) = \frac{\dfrac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} - 1}{\dfrac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} + 1}$$

Or, reducing the algebra, we have:

$$(1-\nu) = \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} - \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} \qquad \text{(A-28)}$$

Solving Eq. A-28 for the ν-parameter, we have

$$\begin{aligned}\nu &= 1 - \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} - \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} \\ &= \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} - \frac{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} - \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}\end{aligned} \qquad \text{(A-29)}$$

Or, reducing terms in Eq. A-29, we obtain:

$$\nu = \frac{2\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\dfrac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}} \qquad \text{(A-30)}$$

We note if the mobility function is constant, then Eq. A-30 reduces to unity, and Eq. A-28 reduces to zero — *which is the result for the single-phase, slightly compressible liquid case.*

Appendix B: Derivation of a General Cubic Inflow Performance Relationship (*IPR*) for Solution Gas-Drive Reservoirs Using a Quadratic Model for the Oil Mobility Function (Alternate Approach to Fetkovich)

In this case we use a quadratic model to represent the oil-phase mobility function. This model is given as:

$$\left[\frac{k_o}{\mu_o B_o}\right]_{\bar{p}} = f(\bar{p}) = a + 2b\bar{p} + 3c\bar{p}^2 \quad \text{(B-1)}$$

We utilize the definition of the oil-phase pseudopressure for this case, which is given by:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp \quad \text{(B-2)}$$

Substituting Eq. B-1 into Eq. B-2 and completing the required integration, we obtain:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \left[(a\bar{p} + b\bar{p}^2 + c\bar{p}^3) - (ap_{base} + bp_{base}^2 + cp_{base}^3)\right] \quad \text{(B-3)}$$

For the oil pseudopressure function, the generalized definition of the "*IPR*"-type formulation ($q_o/q_{o,\max}$) is given as:

$$\frac{q_o}{q_{o,\max}} = \frac{p_{po}(\bar{p}) - p_{po}(p_{wf})}{p_{po}(\bar{p}) - p_{po}(p_{wf} = 0)} \quad \text{(B-4)}$$

Substituting Eq. B-3 into Eq. B-4, gives us:

$$A = (a\bar{p} + b\bar{p}^2 + c\bar{p}^3)$$
$$B = (ap_{base} + bp_{base}^2 + cp_{base}^3)$$
$$C = (ap_{wf} + bp_{wf}^2 + cp_{wf}^3)$$
$$D = (a(0) + b(0)^2 + c(0)^3)$$
$$\frac{q_o}{q_{o,\max}} = \frac{[A-B]-[C-B]}{[A-B]-[D-B]} \quad \text{(B-5)}$$

Cancelling like terms, we obtain:

$$\frac{q_o}{q_{o,\max}} = \frac{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3) - (ap_{wf} + bp_{wf}^2 + cp_{wf}^3)}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3)} \quad \text{(B-6)}$$

Expanding this relation gives:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{ap_{wf}}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3)} - \frac{bp_{wf}^2}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3)} - \frac{cp_{wf}^3}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3)} \quad \text{(B-7)}$$

Writing Eq. B-7 in terms of the "*IPR*" variable $(p_{wf}/\bar{p})$, we have:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \frac{b}{a}\bar{p} + \frac{c}{a}\bar{p}^2)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{a}{b}\frac{1}{\bar{p}} + 1 + \frac{c}{b}\bar{p})}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{1}{(\frac{a}{c}\frac{1}{\bar{p}^2} + \frac{b}{c}\frac{1}{\bar{p}} + 1)}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] \quad \text{(B-8)}$$

Defining $\tau = b/a$, $\beta = c/a$ and $\beta/\tau = c/b$

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{1}{\tau}\frac{1}{\bar{p}} + 1 + \frac{\beta}{\tau}\bar{p})}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{1}{(\frac{1}{\beta}\frac{1}{\bar{p}^2} + \frac{\tau}{\beta}\frac{1}{\bar{p}} + 1)}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] \quad \text{(B-9)}$$

Upon algebraic manipulation Eq. B-9 can be reduced to:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{\tau\bar{p}}{(1 + \tau\bar{p} + \beta\bar{p}^2)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{\beta\bar{p}^2}{(1 + \tau\bar{p} + \beta\bar{p}^2)}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] \quad \text{(B-10)}$$

For this case we define the "lumped parameter," ν, as:

$$\nu = \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2)} \text{ or } \frac{1}{(1 + \frac{b}{a}\bar{p} + \frac{c}{a}\bar{p}^2)} \quad \text{(B-11)}$$

Upon algebraic manipulation, Eq. B-10 can be written as:

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\bar{p}}\right] - \nu\tau\bar{p}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \nu\beta\bar{p}^2\left[\frac{p_{wf}^3}{\bar{p}^3}\right] \quad \text{(B-12)}$$

In Eq. B-12, the ν, τ, and β terms are defined coefficients that contain the characteristic mobility function.

Appendix C: Derivation of a General Quartic Inflow Performance Relationship (*IPR*) for Solution Gas-Drive Reservoirs Using a Cubic Model for the Oil Mobility Function (Alternate Approach to Fetkovich)

In this case we use a cubic model to represent the oil-phase mobility function. This model is given as:

$$\left[\frac{k_o}{\mu_o B_o}\right]_p = f(p) = a + 2b\bar{p} + 3c\bar{p}^2 + 4d\bar{p}^3 \quad \text{(C-1)}$$

We utilize the definition of the oil-phase pseudopressure for this case, which is given by:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp \quad \text{(C-2)}$$

Or, completing the integration, we obtain:

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \times \left[\begin{array}{l}(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4) \\ -(ap_{base} + bp_{base}^2 + cp_{base}^3 + dp_{base}^4)\end{array}\right] \quad \text{(C-3)}$$

For the oil pseudopressure function, the generalized definition of the "*IPR*"-type formulation ($q_o/q_{o,\max}$) is given as:

$$\frac{q_o}{q_{o,\max}} = \frac{p_{po}(\bar{p}) - p_{po}(p_{wf})}{p_{po}(\bar{p}) - p_{po}(p_{wf}=0)} \quad \text{(C-4)}$$

Substituting Eq. C-3 into Eq. C-4, we have:

$$A = (a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)$$

$$B = (ap_{base} + bp_{base}^2 + cp_{base}^3 + dp_{base}^4)$$

$$C = (ap_{wf} + bp_{wf}^2 + cp_{wf}^3 + dp_{wf}^4)$$

$$D = (a(0) + b(0)^2 + c(0)^3 + d(0)^4)$$

$$\frac{q_o}{q_{o,\max}} = \frac{[A-B]-[C-B]}{[A-B]-[D-B]} \quad \text{(C-5)}$$

Recalling the generalized definition of the "*IPR*"-type formulation ($q_o/q_{o,\max}$) for the oil pseudopressure, Eq. (C-2), and canceling like terms, we obtain:

$$\frac{q_o}{q_{o,\max}} = \frac{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4) - (ap_{wf} + bp_{wf}^2 + cp_{wf}^3 + dp_{wf}^4)}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)} \quad \text{(C-6)}$$

Dividing through Eq. C-6 by $(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)$ gives us the following result:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{a\,p_{wf}}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)} - \frac{b\,p_{wf}^2}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)} - \frac{c\,p_{wf}^3}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)} - \frac{d\,p_{wf}^4}{(a\bar{p} + b\bar{p}^2 + c\bar{p}^3 + d\bar{p}^4)} \quad \text{(C-7)}$$

Writing Eq. C-7 in terms of the "*IPR*" variable $(p_{wf}/\bar{p})$, we have:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \frac{b}{a}\bar{p} + \frac{c}{a}\bar{p}^2 + \frac{d}{a}\bar{p}^3)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{a}{b}\frac{1}{\bar{p}} + 1 + \frac{c}{b}\bar{p} + \frac{d}{b}\bar{p}^2)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{1}{(\frac{a}{c}\frac{1}{\bar{p}^2} + \frac{b}{c}\frac{1}{\bar{p}} + 1 + \frac{d}{c}\bar{p})}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] - \frac{1}{(\frac{a}{d}\frac{1}{\bar{p}^3} + \frac{b}{d}\frac{1}{\bar{p}^2} + \frac{c}{d}\frac{1}{\bar{p}} + 1)}\left[\frac{p_{wf}^4}{\bar{p}^4}\right] \quad \text{(C-8)}$$

As done before, defining $\tau = b/a$, $\beta = c/a$, $\eta = d/a$, $\beta/\tau = c/b$, $\eta/\tau = d/b$ and $\eta/\beta = d/c$, we can rewrite Eq. C-8 in terms of these paramaters as:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{1}{(\frac{1}{\tau}\frac{1}{\bar{p}} + 1 + \frac{\beta}{\tau}\bar{p} + \frac{\eta}{\tau}\bar{p}^2)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{1}{(\frac{1}{\beta}\frac{1}{\bar{p}^2} + \frac{\tau}{\beta}\frac{1}{\bar{p}} + 1 + \frac{\eta}{\beta}\bar{p})}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] - \frac{1}{(\frac{1}{\eta}\frac{1}{\bar{p}^3} + \frac{\tau}{\eta}\frac{1}{\bar{p}^2} + \frac{\beta}{\eta}\frac{1}{\bar{p}} + 1)}\left[\frac{p_{wf}^4}{\bar{p}^4}\right] \quad \text{(C-9)}$$

Upon algebraic manipulation, Eq. C-9 can be written as:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)}\left[\frac{p_{wf}}{\bar{p}}\right] - \frac{\tau\bar{p}}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)}\left[\frac{p_{wf}^2}{\bar{p}^2}\right] - \frac{\beta\bar{p}^2}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)}\left[\frac{p_{wf}^3}{\bar{p}^3}\right] - \frac{\eta\bar{p}^3}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)}\left[\frac{p_{wf}^4}{\bar{p}^4}\right] \quad \text{(C-10)}$$

We define the "lumped parameter," ν, for this case as:

$$\nu = \frac{1}{(1 + \tau\bar{p} + \beta\bar{p}^2 + \eta\bar{p}^3)} \text{ or } \frac{1}{(1 + \frac{b}{a}\bar{p} + \frac{c}{a}\bar{p}^2 + \frac{d}{a}\bar{p}^3)} \quad \text{(C-11)}$$

Inserting the "lumped parameter," ν in Eq. C-10:

$$\frac{q_o}{q_{o,\max}}=1-\nu\left[\frac{p_{wf}}{\overline{p}}\right]-\nu\tau\,\overline{p}\left[\frac{p_{wf}^2}{\overline{p}^2}\right]-\nu\beta\,\overline{p}^2\left[\frac{p_{wf}^3}{\overline{p}^3}\right]-\nu\eta\,\overline{p}^3\left[\frac{p_{wf}^4}{\overline{p}^4}\right]$$

.... (C-12)

In Eq. C-12, the ν, τ, β and η terms are defined coefficients that contain the characteristic mobility function.

Appendix D: Derivation of the Quartic Inflow Performance Relationship (*IPR*) for Solution Gas-Drive Reservoirs Using the *Proposed Cubic* (*Characteristic*) *Model* for the Oil Mobility Function

For reference we present the characteristic model for the oil mobility function according to our normalized variables as:

$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\overline{p}}-[k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i}-[k_o/(\mu_o B_o)]_{p_{abn}}}\right]=$$
$$1-\zeta\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^2-2(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$
$$(\zeta\le 1)$$

.... (D-1)

We rearrange Eq. D-1 (*i.e*. the characteristic model) in terms of the oil mobility function evaluated at any average reservoir pressure as:

$$\begin{aligned}f(\overline{p})-f(p_{abn})&\\ &=\frac{f(p_i)-f(p_{abn})}{p_i-p_{abn}}\zeta\,(\overline{p}-p_{abn})\\ &\quad-\frac{f(p_i)-f(p_{abn})}{(p_i-p_{abn})^2}(1-\zeta)(\overline{p}-p_{abn})^2\\ &\quad+\frac{f(p_i)-f(p_{abn})}{(p_i-p_{abn})^3}2(1-\zeta)(\overline{p}-p_{abn})^3\end{aligned}$$ .... (D-2)

Where

$f(\overline{p})=[k_o/(\mu_o B_o)]_{\overline{p}}$,

$f(p_i)=[k_o/(\mu_o B_o)]_{p_i}$,

$f(p_{abn})=[k_o/(\mu_o B_o)]_{p_{abn}}$

Recalling the general cubic model to represent the oil-phase mobility function which was is given in Eq. C-1 as:

$$\left[\frac{k_o}{\mu_o B_o}\right]_{\overline{p}}$$
$$=f(\overline{p}-p_{abn})=a+2b(\overline{p}-p_{abn})+3c(\overline{p}-p_{abn})^2+4d(\overline{p}-p_{abn})^3$$

.... (C-1)

Eq D-2 implies that the parameter a in Eq. C-1 (the intercept where average reservoir pressure is equal to zero) will equal to the value of the oil mobility at the abandonment pressure for our purposes. Recalling Eq. C-7:

$$\begin{aligned}\frac{q_o}{q_{o,\max}}=1&-\frac{a\,p_{wf}}{(a\overline{p}+b\overline{p}^2+c\overline{p}^3+d\overline{p}^4)}\\ &-\frac{b\,p_{wf}^2}{(a\overline{p}+b\overline{p}^2+c\overline{p}^3+d\overline{p}^4)}\\ &-\frac{c\,p_{wf}^3}{(a\overline{p}+b\overline{p}^2+c\overline{p}^3+d\overline{p}^4)}\\ &-\frac{d\,p_{wf}^4}{(a\overline{p}+b\overline{p}^2+c\overline{p}^3+d\overline{p}^4)}\end{aligned}$$

.... (D-3)

Further manipulating Eq. D-3 in terms of the "*IPR*" variable $(p_{wf}/\overline{p})$, we have:

$$\begin{aligned}\frac{q_o}{q_{o,\max}}=1&-\frac{1}{(1+\frac{b}{a}\overline{p}+\frac{c}{a}\overline{p}^2+\frac{d}{a}\overline{p}^3)}\left[\frac{p_{wf}}{\overline{p}}\right]\\ &-\frac{1}{(\frac{a}{b}\frac{1}{\overline{p}}+1+\frac{c}{b}\overline{p}+\frac{d}{b}\overline{p}^2)}\left[\frac{p_{wf}^2}{\overline{p}^2}\right]\\ &-\frac{1}{(\frac{a}{c}\frac{1}{\overline{p}^2}+\frac{b}{c}\frac{1}{\overline{p}}+1+\frac{d}{c}\overline{p})}\left[\frac{p_{wf}^3}{\overline{p}^3}\right]\\ &-\frac{1}{(\frac{a}{d}\frac{1}{\overline{p}^3}+\frac{b}{d}\frac{1}{\overline{p}^2}+\frac{c}{d}\frac{1}{\overline{p}}+1)}\left[\frac{p_{wf}^4}{\overline{p}^4}\right]\end{aligned}$$

.... (D-4)

Recalling the definitions, $\tau=b/a$, $\beta=c/a$, $\eta=d/a$, $\beta/\tau=c/b$, $\eta/\tau=d/b$ and $\eta/\beta=d/c$, Eq. D-4 can be written as:

$$\begin{aligned}\frac{q_o}{q_{o,\max}}=1&-\frac{1}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)}\left[\frac{p_{wf}}{\overline{p}}\right]\\ &-\frac{1}{(\frac{1}{\tau}\frac{1}{\overline{p}}+1+\frac{\beta}{\tau}\overline{p}+\frac{\eta}{\tau}\overline{p}^2)}\left[\frac{p_{wf}^2}{\overline{p}^2}\right]\\ &-\frac{1}{(\frac{1}{\beta}\frac{1}{\overline{p}^2}+\frac{\tau}{\beta}\frac{1}{\overline{p}}+1+\frac{\eta}{\beta}\overline{p})}\left[\frac{p_{wf}^3}{\overline{p}^3}\right]\\ &-\frac{1}{(\frac{1}{\eta}\frac{1}{\overline{p}^3}+\frac{\tau}{\eta}\frac{1}{\overline{p}^2}+\frac{\beta}{\eta}\frac{1}{\overline{p}}+1)}\left[\frac{p_{wf}^4}{\overline{p}^4}\right]\end{aligned}$$

.... (D-5)

Upon algebraic manipulation, we have the following form below:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)}\left[\frac{p_{wf}}{\overline{p}}\right] - \frac{\tau\,\overline{p}}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)}\left[\frac{p_{wf}^2}{\overline{p}^2}\right] - \frac{\beta\,\overline{p}^2}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)}\left[\frac{p_{wf}^3}{\overline{p}^3}\right] - \frac{\eta\,\overline{p}^3}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)}\left[\frac{p_{wf}^4}{\overline{p}^4}\right]$$

.... (D-6)

Recalling the definition of the "lumped parameter," ν:

$$\nu = \frac{1}{(1+\tau\,\overline{p}+\beta\,\overline{p}^2+\eta\,\overline{p}^3)} \text{ or } \frac{1}{(1+\frac{b}{a}\overline{p}+\frac{c}{a}\overline{p}^2+\frac{d}{a}\overline{p}^3)} \quad \text{..... (C-11)}$$

Inserting the "lumped parameter," ν in Eq. D-6:

$$\frac{q_o}{q_{o,\max}} = 1-\nu\left[\frac{p_{wf}}{\overline{p}}\right]-\nu\tau\,\overline{p}\left[\frac{p_{wf}^2}{\overline{p}^2}\right]-\nu\beta\,\overline{p}^2\left[\frac{p_{wf}^3}{\overline{p}^3}\right]-\nu\eta\,\overline{p}^3\left[\frac{p_{wf}^4}{\overline{p}^4}\right]$$

.... (D-7)

Referring to the proposed characteristic model for the oil mobility function, the coefficients in Eq. C-1 correspond to the following:

$$a = f(p_{abn})$$

$$b = \frac{f(p_i)-f(p_{abn})}{2(p_i-p_{abn})}\zeta$$

$$c = \frac{f(p_i)-f(p_{abn})}{3(p_i-p_{abn})^2}(\zeta-1)$$

$$d = \frac{f(p_i)-f(p_{abn})}{4(p_i-p_{abn})^3}2(1-\zeta)$$

.... (D-8)

Combining the previous definitions of, $\tau = b/a$, $\beta = c/a$, $\eta = d/a$, $\beta/\tau = c/b$, $\eta/\tau = d/b$ and $\eta/\beta = d/c$, with the coefficients given in Eq. D-8, we have:

$$\tau = \frac{[f(p_i)-f(p_{abn})]}{2(p_i-p_{abn})}\frac{\zeta}{f(p_{abn})}$$

$$\beta = \frac{[f(p_i)-f(p_{abn})]}{3(p_i-p_{abn})^2}\frac{(\zeta-1)}{f(p_{abn})}$$

$$\eta = \frac{[f(p_i)-f(p_{abn})]}{4(p_i-p_{abn})^3}\frac{2(1-\zeta)}{\zeta\, f(p_i)}$$

$$\beta/\tau = \frac{2}{3}\frac{(\zeta-1)}{\zeta}\frac{1}{(p_i-p_{abn})}$$

$$\eta/\tau = \frac{(1-\zeta)}{\zeta}\frac{1}{(p_i-p_{abn})^2}$$

$$\eta/\beta = \frac{-3}{2}\frac{1}{(p_i-p_{abn})}$$

.... (D-9)

Subsituting the obtained values above in the quartic "*IPR*" equation (Eq. D-7), we have the final form of the "*IPR*" equation in terms of the *charactarestic parameter*, *initial pressure*, *abandonment pressure* and the *average reservoir pressure*:

SPE 110821

Inflow Performance Relationship (IPR) for Solution Gas-Drive Reservoirs — Analytical Considerations

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Presentation Outline

- Rationale for This Work (*IPR* and mobility concepts)
- Due Diligence — *IPR* Approaches
- What's New About this Work
- *IPR* Formulation for Solution Gas-Drive Systems
- Characteristic Behavior of Solution Gas-Drive Reservoirs
- Summary and Conclusions

Problem: Generalized Flow Theory for *PSS* Flow (*IPR*)

***Issues*:**

— Is the Vogel IPR form empirical, analytical, or something in between?

— How do we establish an analytical basis for the case of a well in a solution gas-drive reservoir?

$$\frac{q}{q_{x,\max}} = 1 - v_x \left[\frac{p_{wf}}{\overline{p}}\right] - (1 - v_x)\left[\frac{p_{wf}}{\overline{p}}\right]^2$$

$$(x = \text{phase}, (\text{oil, gas, and water}))$$

Rationale For This Work

1. Establish the validity of the quadratic *IPR* relation:

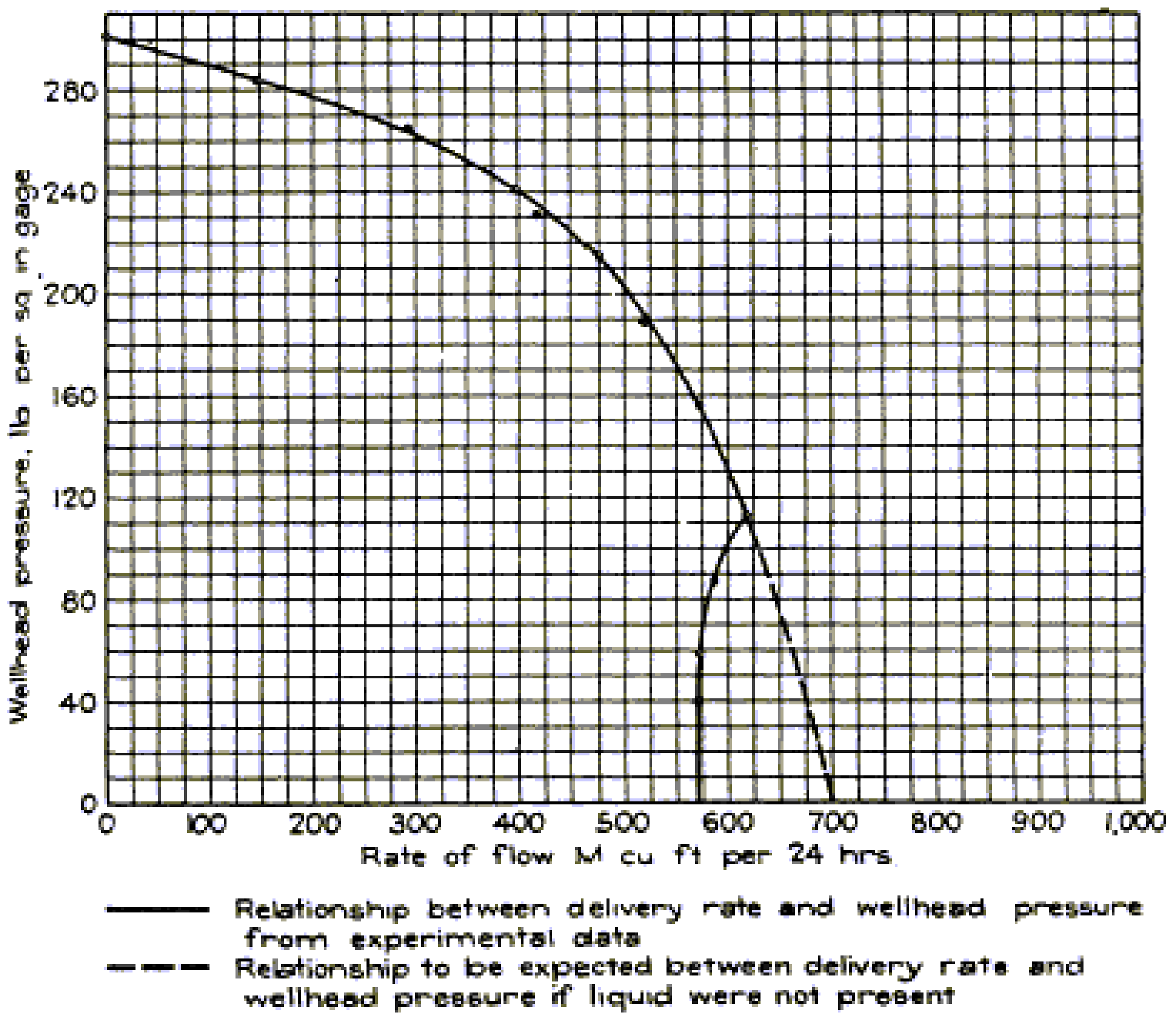
$$\frac{q}{q_{o,\max}} = 1 - v_o \left[\frac{p_{wf}}{\overline{p}}\right] - (1 - v_o)\left[\frac{p_{wf}}{\overline{p}}\right]^2 \qquad 0 \le v_o \le 1$$

2. Establish the character of the oil mobility function:

$$\left[1 - \frac{[k_o/(\mu_o B_o)]_{\overline{p}}}{[k_o/(\mu_o B_o)]_{p_i}}\right] = \qquad \left(\text{Assuming}\,[k_o/(\mu_o B_o)]_{\overline{p}=p_{abn}} = 0\right)$$

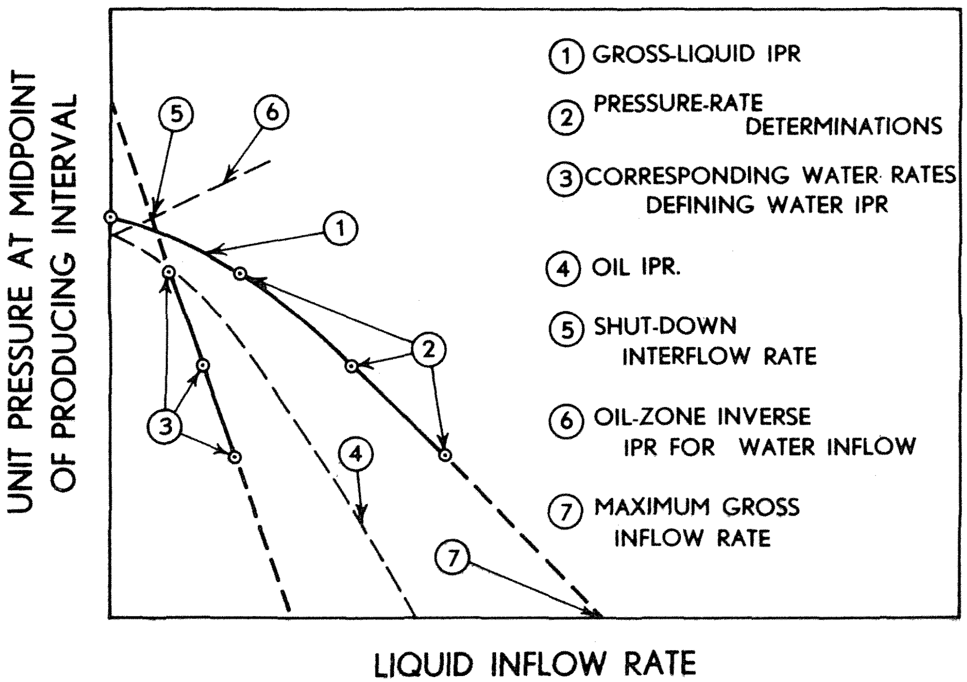
$$1 - \zeta\left[\frac{\overline{p}}{p_i}\right] + (1-\zeta)\left[\frac{\overline{p}}{p_i}\right]^2 - 2(1-\zeta)\left[\frac{\overline{p}}{p_i}\right]^3 \quad (\zeta \le 1)$$

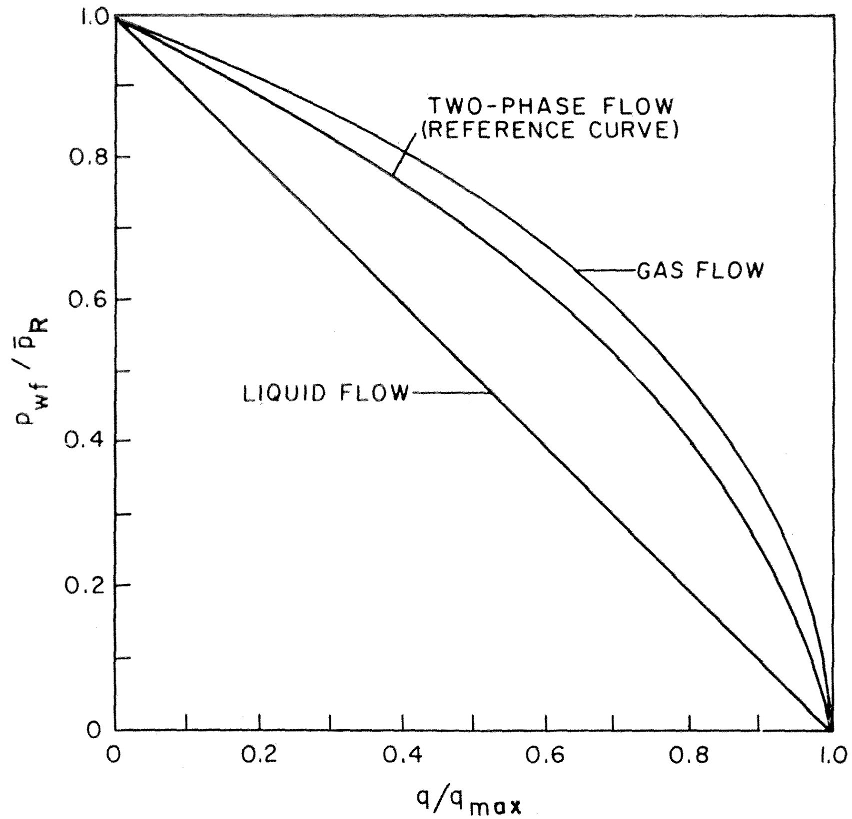
Due Diligence: *Previous IPR Approaches*



- **Early "*IPR* Plot" for a Gas Well: Rawlins and Schellhardt (1935)**
  - **Quadratic relationship between p_{tf} and q_g (circa 1935).**
  - **Early attempt to quantify and predict future performance.**

Due Diligence: *Previous IPR Approaches*

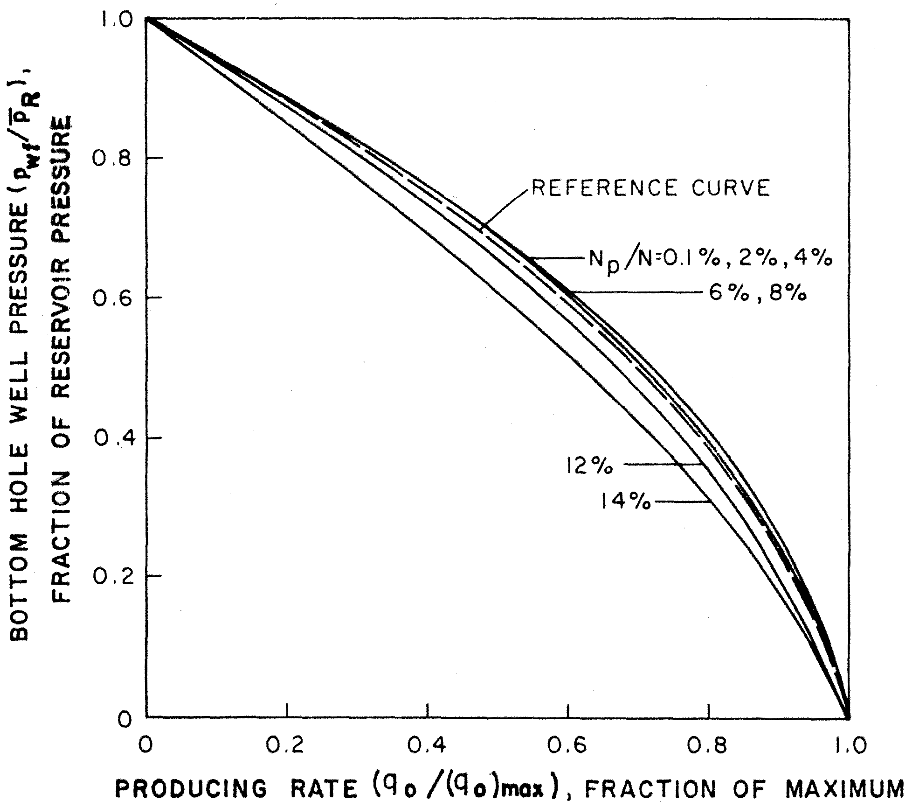




- Early "Inflow Plot," an attempt to correlate well rate and pressure behavior — and to establish the maximum flowrate [Gilbert (1954)].
- *IPR* "comparison" — liquid (oil), gas, and "two-phase" (solution gas-drive) cases presented to illustrate comparative behavior [Vogel (1968)].

- **Inflow Performance Relationship (*IPR*):**
  - Correlate performance and estimate maximum flowrate.
  - Individual phases require separate correlations.

<u>Due Diligence</u>: *Previous IPR Approaches*



- **<u>Vogel Correlation</u>: (Statistical)**

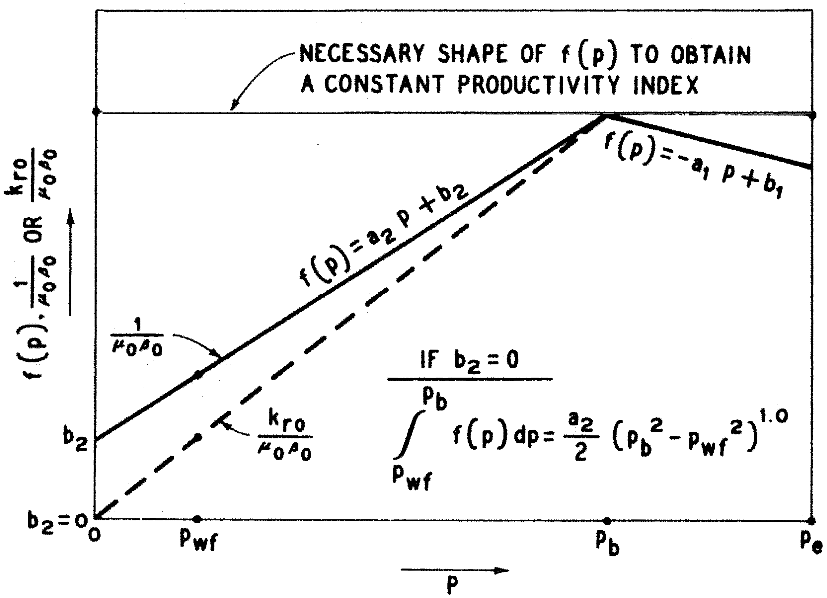
$$\frac{q}{q_{o,\max}} = 1 - 0.2\left[\frac{p_{wf}}{\bar{p}}\right] - 0.8\left[\frac{p_{wf}}{\bar{p}}\right]^2$$

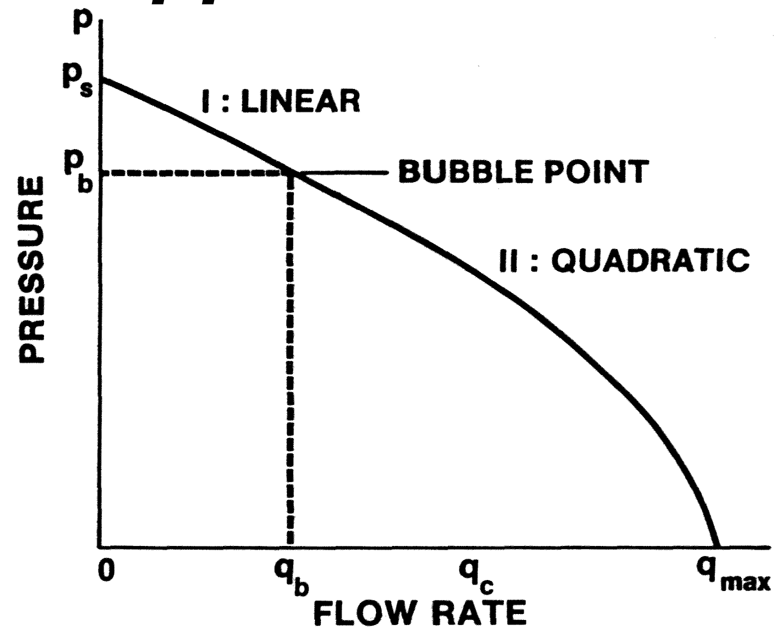
***<u>Comment</u>*:**
— ***The Vogel IPR correlation is well-established as a performance prediction relation.***
— ***The Vogel correlation is "derived" from exhaustive reservoir simulation cases.***

- ***IPR behavior is dependent on the depletion stage*** **(*i.e.*, the level of reservoir depletion). No single-trend correlation of *IPR* behavior is possible.**

- **<u>Vogel *IPR* Correlation</u>: *Solution Gas-Drive Behavior* (1968)**
  - **Derived as a statistical correlation from simulation cases.**
  - **No "theoretical" basis — intuitive correlation ($q_{o,max}$ and p_{avg}).**

Due Diligence: *Previous IPR Approaches*





- **Fetkovich *IPR***: Semi-empirical, assumes linear mobility function.

$$\frac{q}{q_{o,\max}} = \left[1 - \left[\frac{p_{wf}}{\bar{p}}\right]^2\right]^n$$

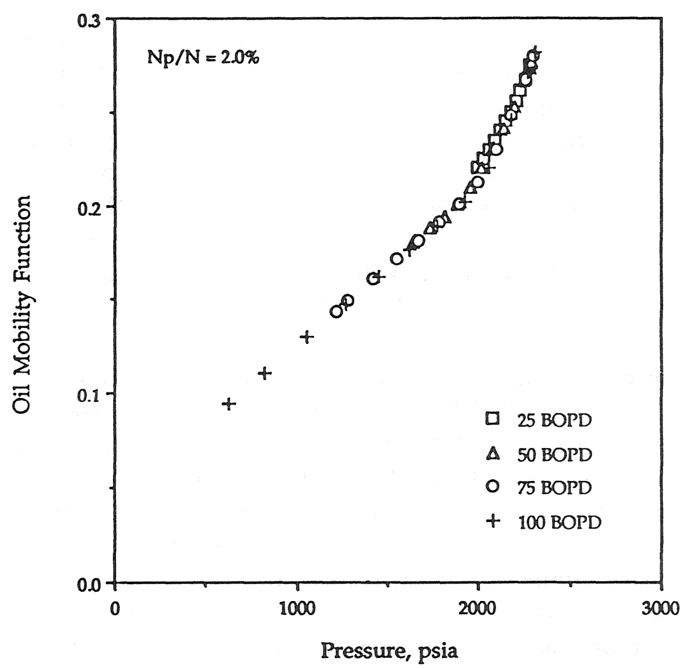
- **Richardson *et al***: Empirical, used Vogel form and generalized the coefficient.

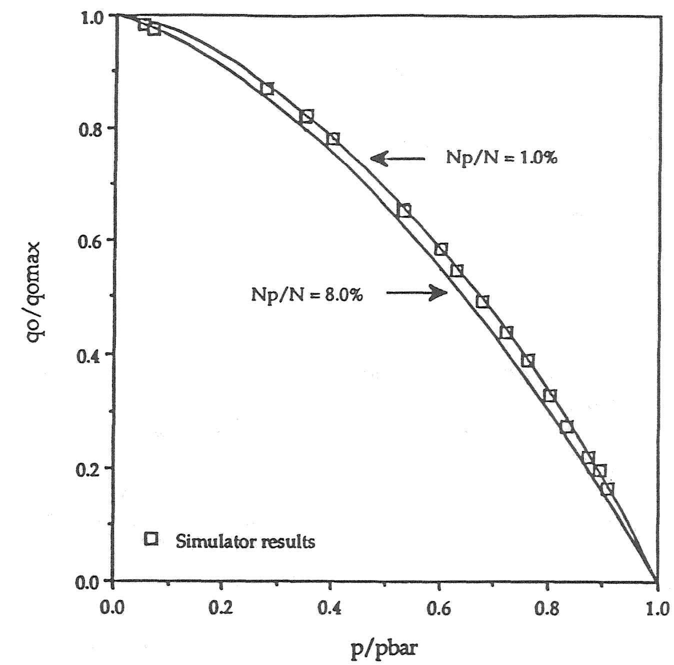
$$\frac{q}{q_{o,\max}} = 1 - \nu_x\left[\frac{p_{wf}}{\bar{p}}\right] - (1-\nu_x)\left[\frac{p_{wf}}{\bar{p}}\right]^2$$

$$(x = \text{phase}, (\text{oil, gas, and water}))$$

- **Other *IPR* Correlations**: Fetkovich (1973) and Richardson (1982)
  - Fetkovich (1973) assumed linear mobility-pressure relationship.
  - Richardson *et al*. (1982) generalized the "ν"-coefficient.

Due Diligence: *Previous IPR Approaches*



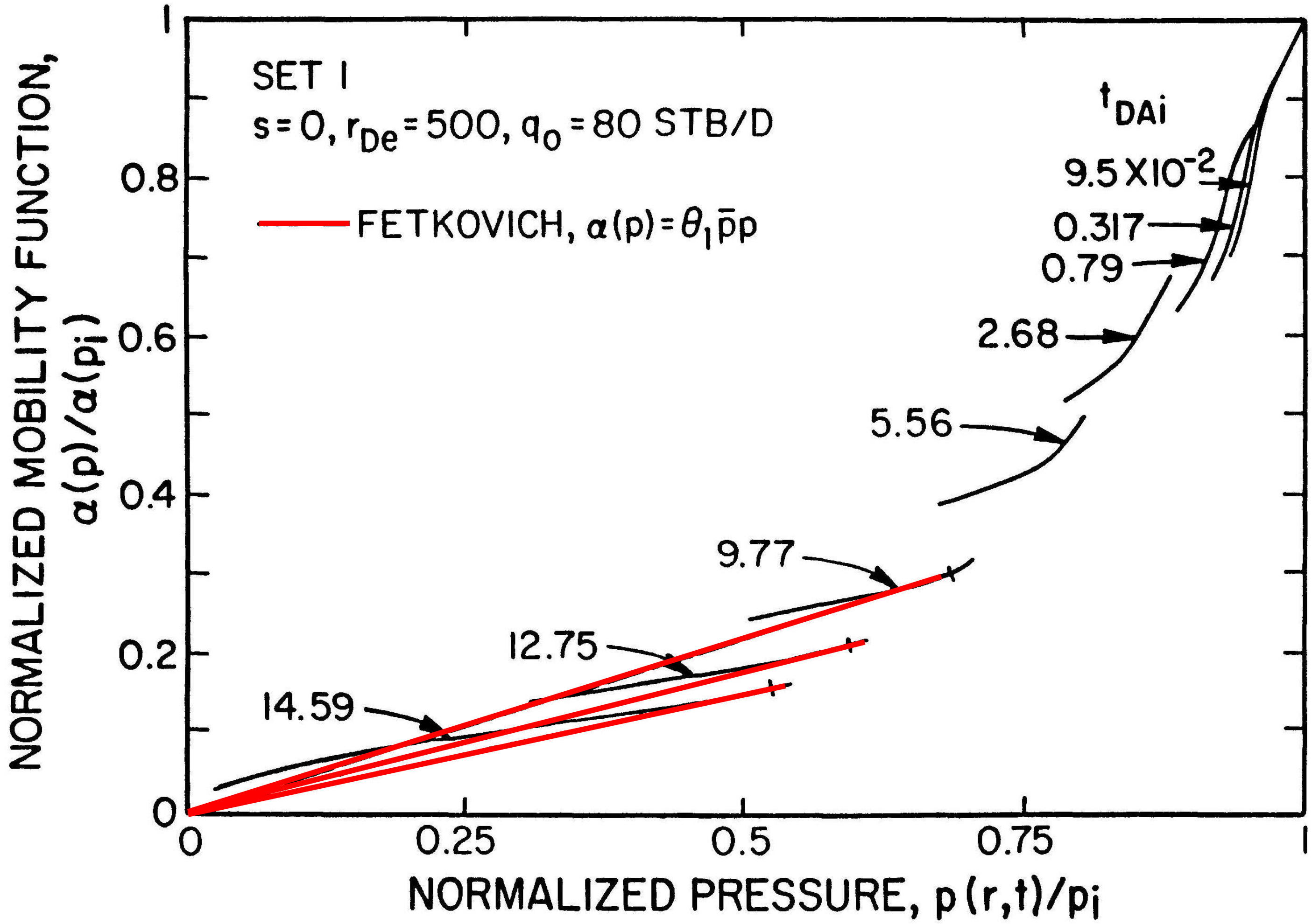


- **Wiggins *et al IPR*: Semi-empirical, mobility represented as polynomial.**
- **Wiggins *et al IPR*: *IPR* performance is quite accurate, but must know complete mobility profile.**

$$\frac{q}{q_{o,\max}} = 1 + a_1\left[\frac{p_{wf}}{\bar{p}}\right] + a_2\left[\frac{p_{wf}}{\bar{p}}\right]^2 + a_3\left[\frac{p_{wf}}{\bar{p}}\right]^3 + \ldots\ldots$$

- **Other *IPR* Correlations: Wiggins *et al* (1996)**
  - **Wiggins *et al* (1996) used a polynomial expansion for mobility.**
  - **Requires complete knowledge of mobility function.**

Due Diligence: *Previous IPR Approaches*



- **Other *IPR* Correlations: Camacho and Raghavan**
  - **Camacho-Raghavan (1991) — mobility versus pressure over time.**
  - **Fetkovich model (and others) only valid at very late times.**
  - **What "inspiration" does this map provide for us?**

What's New About this Work: *The Big Picture*

- Goal — To unify and utilize the Camacho and Raghavan map of mobility behavior to provide a unifying theory (at least an understanding) for the pseudosteady-state performance of solution gas-drive systems.
- In simple language … to establish a characteristic (*i.e.*, simple model) for solution gas-drive behavior.
- We think we have it … mobility is a characteristic function of pressure:

$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\bar{p}}-[k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i}-[k_o/(\mu_o B_o)]_{p_{abn}}}\right]=$$

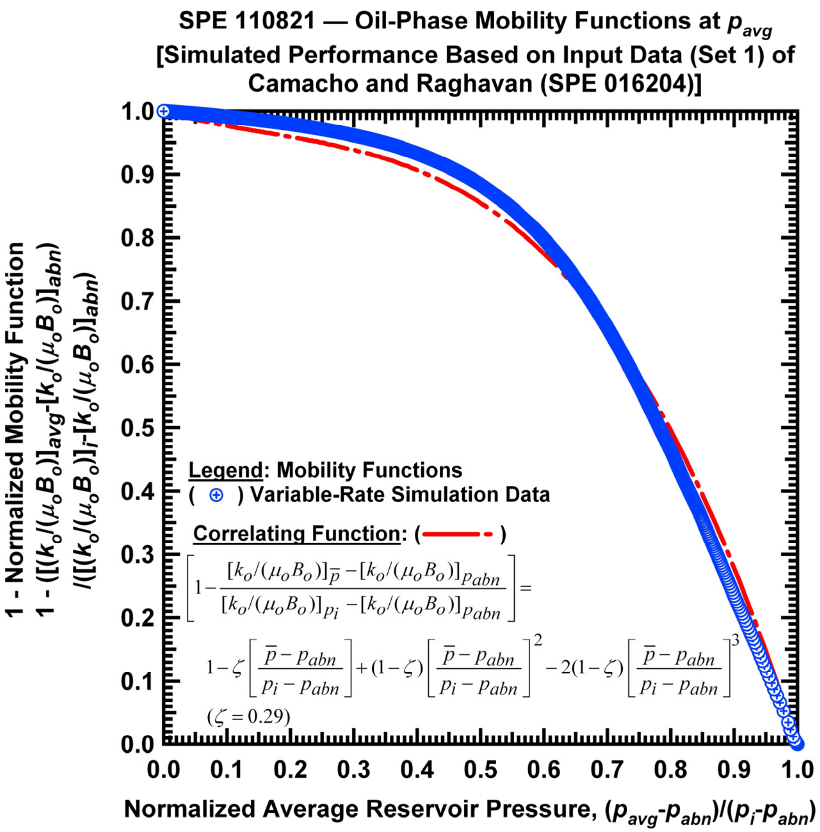
$$1-\zeta\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^2-2(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$

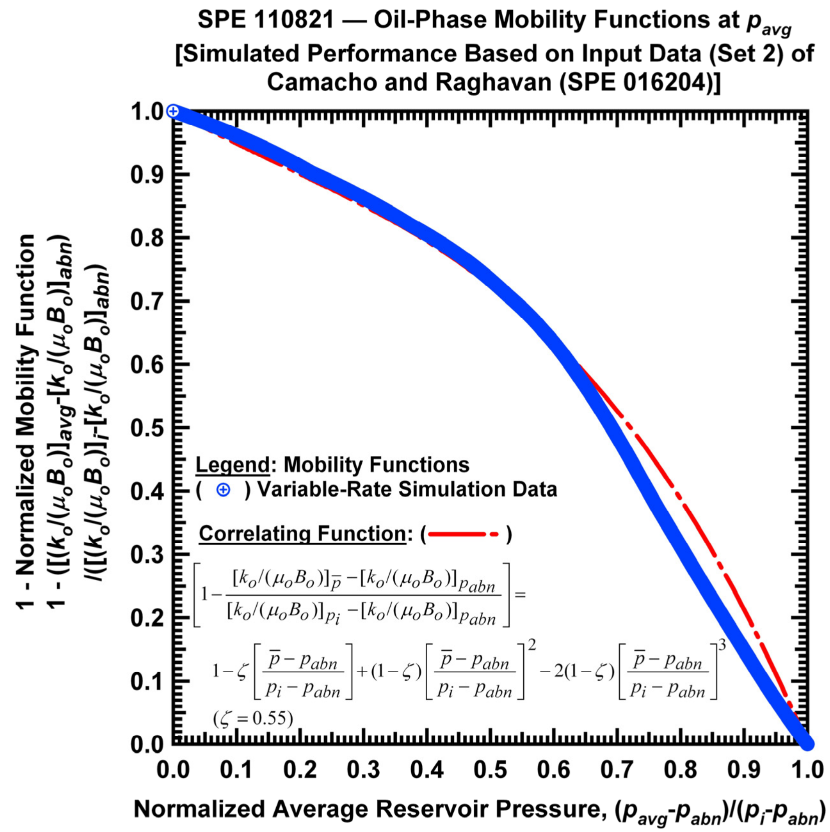
$$(\zeta\le 1)$$

A theory is more impressive the greater the simplicity of its premise, the more different kinds of things it relates and the more extended is its area of applicability.

Albert Einstein

What's New About this Work: *The Big Picture*





- **Validation Case 1**: Camacho-Raghavan "Set 1" — cubic match.
- **Validation Case 2**: Camacho-Raghavan "Set 2" — cubic match.

- **Validation Cases**: Camacho and Raghavan (Data Sets 1 and 2)
  - Entire profile represented by "endpoint" mobilities and ζ.
  - Should the correlating relation be quartic instead of cubic?
  - We have to provide exhaustive validation, … but it should work.

IPR Formulation for Solution-Gas Drive Systems:

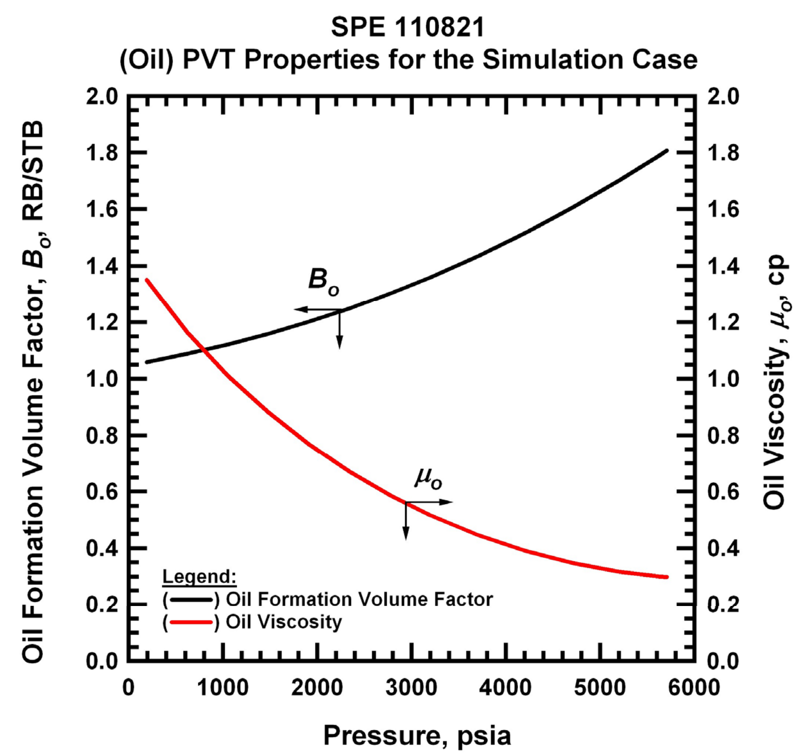
- **The solution gas-drive case requires the use of an oil-phase pseudopressure — diffusivity equation is nonlinear (*i.e.*, μ_o, B_o, k_o are functions of pressure):**
  - **Evinger and Muskat presented a variation of the oil phase pseudopressure equation for steady-state flow in 1942!**
  - **Major issues: $k_o=f(S_o,p)$, $\mu_o=f(p)$, and $B_o=f(p)$,**

Calculation of Productivity Factors for Oil-gas-water Systems in the Steady State

By H. H. Evinger* and M. Muskat*

(New York Meeting, February 1942)

$$\frac{Q_o}{2\pi hk}\log r_e/r = \int_p^{p_e} \frac{k_o/k}{\mu_o\beta}\,dp$$



- **Oil-Phase Pseudopressure:**

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp$$

IPR Formulation for Solution-Gas Drive Systems:

- Camacho and Raghavan (1989) presented the pseudo-steady-state flow model for the oil-phase in a solution gas-drive system as:

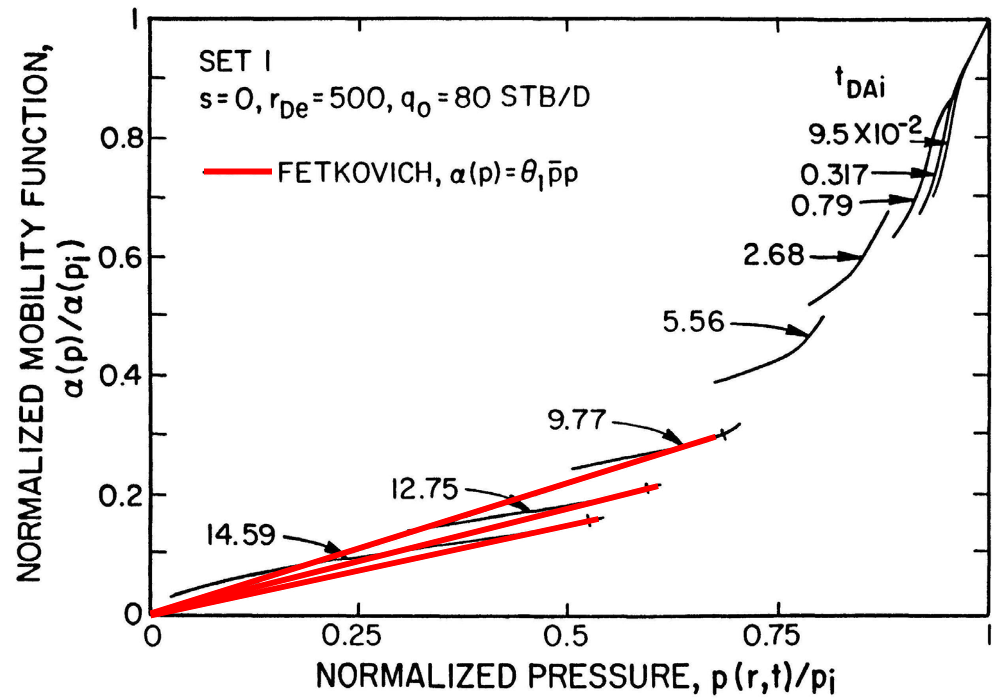
$$q_o = \frac{1}{b_{pss}}[p_{po}(\overline{p}) - p_{po}(p_{wf})] \quad \left[p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} \left[\frac{k_o}{\mu_o B_o}\right] dp \right]$$

Next Steps:

— *In order to "characterize" the performance of solution gas-drive systems, Camacho and Raghavan used numerical simulation.*

— *The "characteristic" variables appear to be normalized mobility and normalized pressure.*

— *The condition at p=0 (or abandonment pressure will have to be addressed).*



IPR Formulation for Solution-Gas Drive Systems:

- The Vogel quadratic *IPR* form is based on the assumption that the mobility profile is linear (specifically, $p<p_b$).

$$[k_o/(\mu_o B_o)]_{\overline{p}} = f(\overline{p}) = a + 2b\,\overline{p}$$

  - Fetkovich (1973) used this formulation to develop his "deliverability" equations for solution gas-drive systems (p^2-form).
  - A "quasi-analytical" derivation of the Vogel *IPR* is given below:

1

$$p_{po}(p) = \left[\frac{\mu_o B_o}{k_o}\right]_{p_n} \int_{p_{base}}^{p} (a + 2bp)\, dp$$

2

$$q_o = \frac{1}{b_{pss}}[p_{po}(\overline{p}) - p_{po}(p_{wf})]$$

3

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{1}{(1+\frac{b}{a}\overline{p})}\left[\frac{p_{wf}}{\overline{p}}\right] - \frac{1}{(1+\frac{a}{b}\frac{1}{\overline{p}})}\left[\frac{p_{wf}}{\overline{p}}\right]^2$$

4

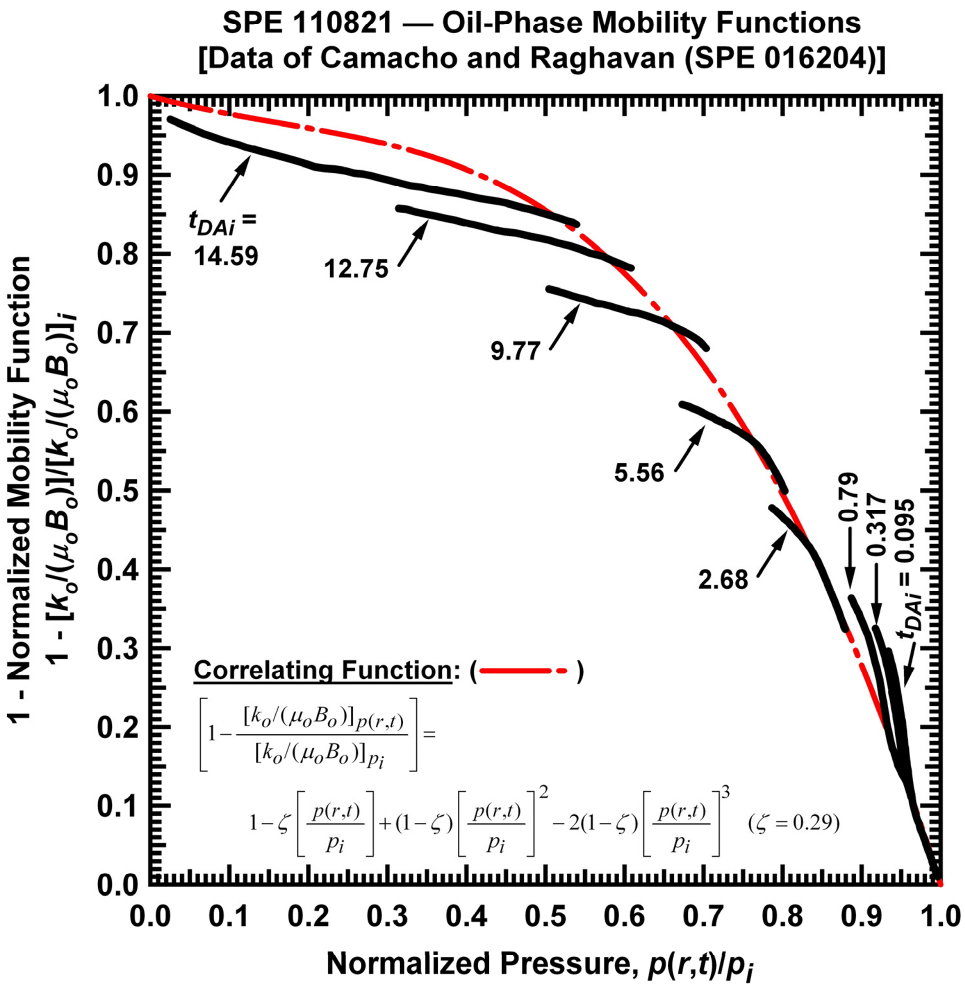
$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{\overline{p}}\right] - (1-v)\left[\frac{p_{wf}}{\overline{p}}\right]^2$$

5

$$v = \frac{2\left[\frac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}{\left[\frac{k_o}{\mu_o B_o}\right]_{\overline{p}} + \left[\frac{k_o}{\mu_o B_o}\right]_{\overline{p}=0}}$$

(v = 0.2 for Vogel *IPR*)

Characteristic Behavior: *Solution Gas-Drive Res.*



- **"Partial" Normalization:**

$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\overline{p}}}{[k_o/(\mu_o B_o)]_{p_i}}\right]=$$

$$1-\zeta\left[\frac{\overline{p}}{p_i}\right]+(1-\zeta)\left[\frac{\overline{p}}{p_i}\right]^2$$

$$-2(1-\zeta)\left[\frac{\overline{p}}{p_i}\right]^3 \quad (\zeta \le 1)$$

- **Characteristic Behavior: Mobility-Pressure**
  - Mobility and pressure functions are normalized.
  - Linear mobility assumption is not valid (except at late times).
  - Uniqueness of mobility signature? (unique, but not universal).

Characteristic Behavior: *Solution Gas-Drive Res.*

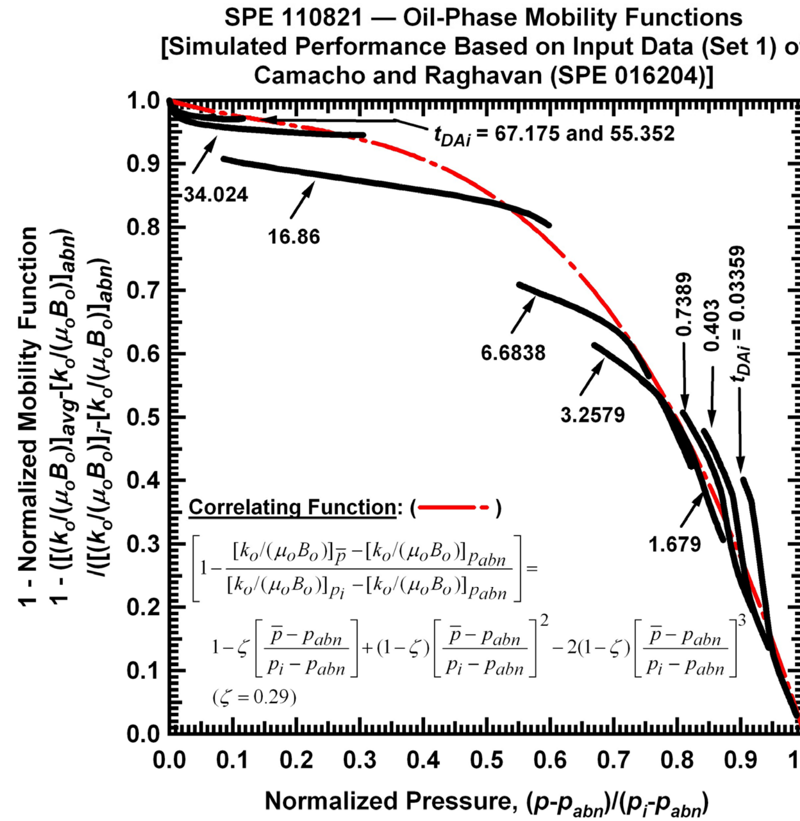
- **Full Normalization:**

$$\left[1-\frac{[k_o/(\mu_oB_o)]_{\overline{p}}-[k_o/(\mu_oB_o)]_{p_{abn}}}{[k_o/(\mu_oB_o)]_{p_i}-[k_o/(\mu_oB_o)]_{p_{abn}}}\right]=$$

$$1-\zeta\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^2$$

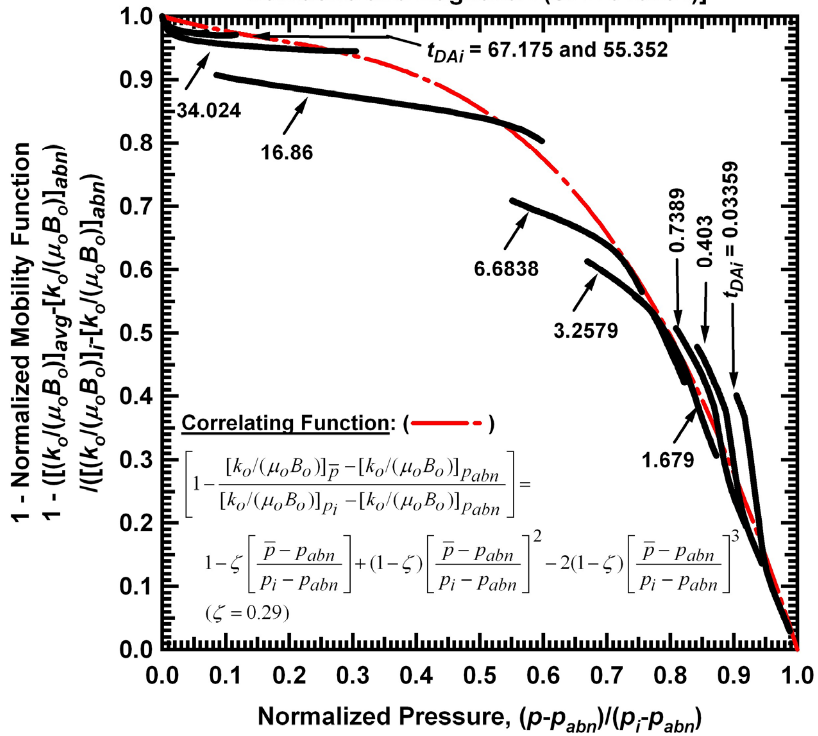
$$-2(1-\zeta)\left[\frac{\overline{p}-p_{abn}}{p_i-p_{abn}}\right]^3 \qquad (\zeta\le 1)$$

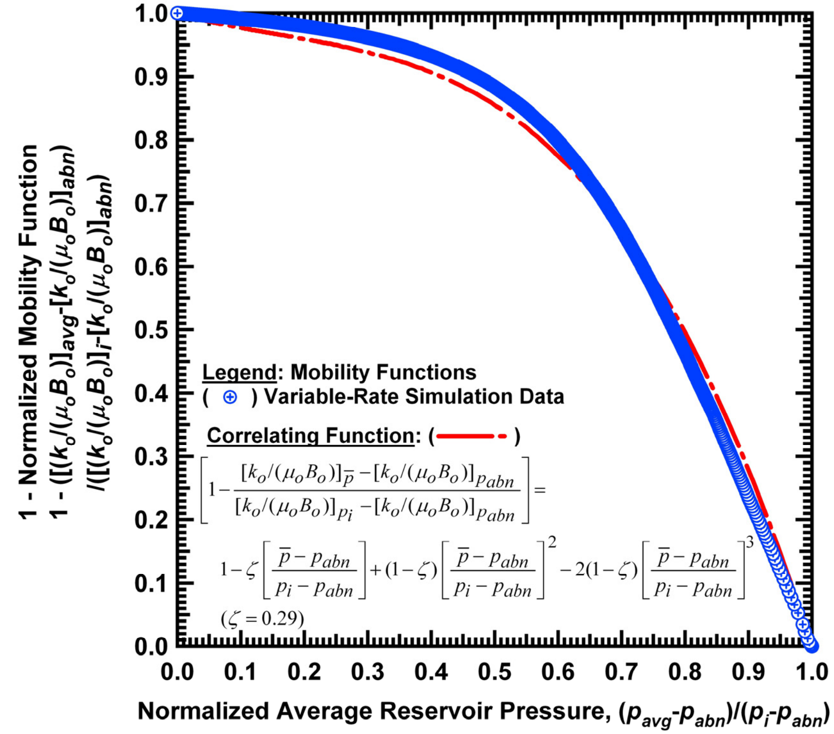
Characteristic parameter (ζ) is unique (i.e., constant) for a given scenario.



- **Characteristic Mobility Parameter (ζ):** ***Correlation Terms***
  - Initial reservoir pressure, p_i.
  - Average reservoir pressure, p_{avg}.
  - Abandonment pressure, p_{abn}.
  - Oil-phase mobility evaluated at p_i.
  - Oil-phase mobility evaluated at p_{abn}.

Characteristic Behavior: *Solution Gas-Drive Res.*



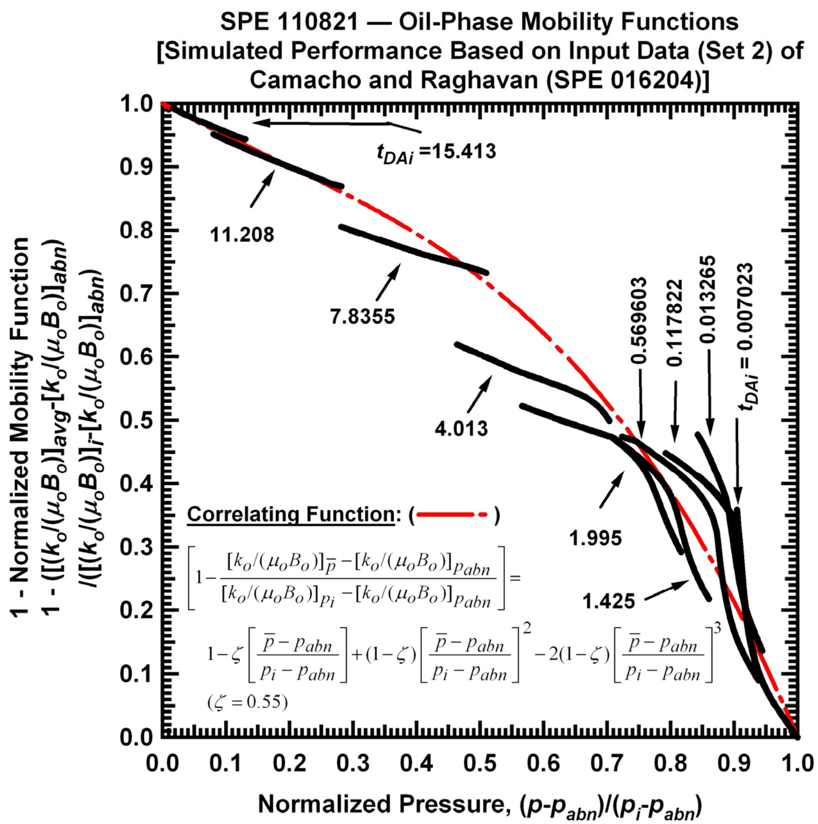


$$\left[1-\frac{[k_o/(\mu_o B_o)]_{\bar{p}}-[k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i}-[k_o/(\mu_o B_o)]_{p_{abn}}}\right]=1-\zeta\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]+(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^2-2(1-\zeta)\left[\frac{\bar{p}-p_{abn}}{p_i-p_{abn}}\right]^3$$

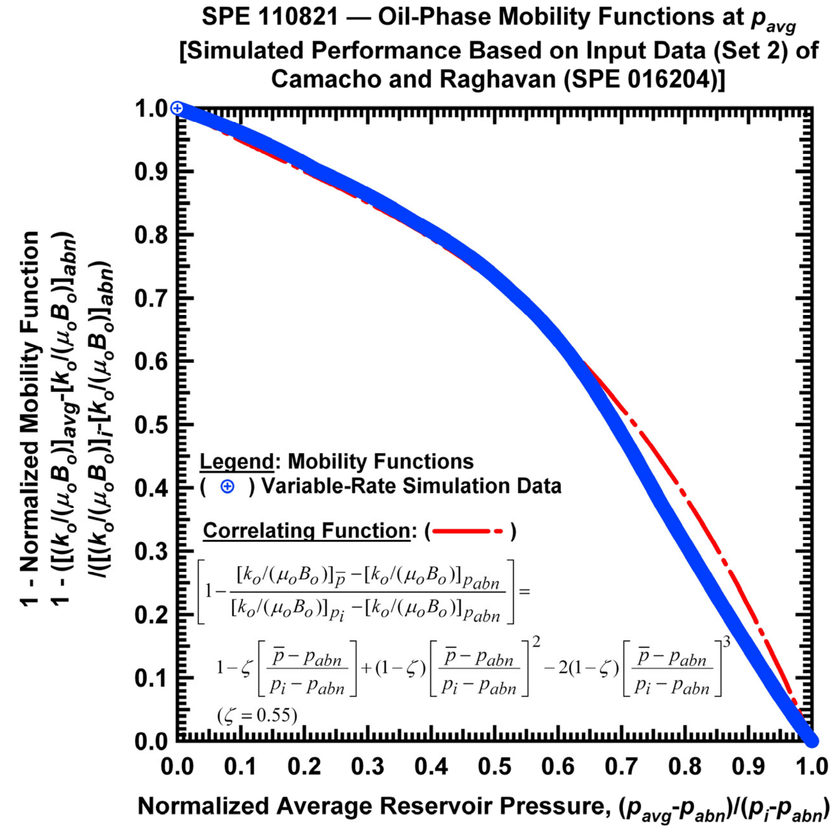
- Normalized mobility function plotted versus normalized *reservoir* pressure.
- Normalized mobility function plotted versus normalized *average* pressure.

- <u>Validation</u>: *Data of Camacho and Raghavan* (1989) — "Set 1"
  - Calibration using "Data Set 1" of Camacho and Raghavan (light oil).
  - Mobility profile at average reservoir pressure — characteristic concept.
  - ζ = 0.29 for this case — good match of the mobility profile.

Characteristic Behavior: *Solution Gas-Drive Res.*



■ Normalized mobility function plotted versus normalized *reservoir* pressure.



■ Normalized mobility function plotted versus normalized *average* pressure.

- **Validation**: *Data of Camacho and Raghavan* (1989) — "Set 2"
  - ■ Calibration using "Data Set 2" of Camacho and Raghavan (heavy oil).
  - ■ Mobility profile at average reservoir pressure — characteristic concept.
  - ■ ζ = 0.55 for this case — good match, but should we use quartic?

IPR Functions …

1. ***CUBIC IPR*** **using** ***QUADRATIC*** **characteristic function:**

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\overline{p}}\right] - \nu\tau\,\overline{p}\left[\frac{p_{wf}^2}{\overline{p}^2}\right] - \nu\beta\,\overline{p}^2\left[\frac{p_{wf}^3}{\overline{p}^3}\right]$$

2. ***QUARTIC IPR*** **using** ***CUBIC*** **characteristic function:**

$$\frac{q_o}{q_{o,\max}} = 1 - \nu\left[\frac{p_{wf}}{\overline{p}}\right] - \nu\tau\,\overline{p}\left[\frac{p_{wf}^2}{\overline{p}^2}\right] - \nu\beta\,\overline{p}^2\left[\frac{p_{wf}^3}{\overline{p}^3}\right] - \nu\eta\,\overline{p}^3\left[\frac{p_{wf}^4}{\overline{p}^4}\right]$$

Where, ν, τ, β, and η are parameters uniquely defined by the *characteristic function*.

Conclusions and Recommendations

- **Summary:**
  - The following characteristic relation was obtained in this work:

$$\left[\frac{[k_o/(\mu_o B_o)]_{\overline{p}} - [k_o/(\mu_o B_o)]_{p_{abn}}}{[k_o/(\mu_o B_o)]_{p_i} - [k_o/(\mu_o B_o)]_{p_{abn}}}\right] = f\left[\zeta, \frac{\overline{p} - p_{abn}}{p_i - p_{abn}}\right] \quad (\zeta \leq 1)$$

  - This formulation was shown to be unique — and to provide a basis for a unified pseudosteady-state flow concept for the case of a solution gas-drive reservoir system.

- **Conclusions:**
  - The characteristic mobility parameter (ζ) uniquely defines the mobility profile for the performance of a solution gas-drive reservoir.
  - The cubic and quartic *IPR* formulations derived using the quadratic and cubic expansions for oil-phase mobility are considered unique as these results were derived based on the concept of the characteristic mobility function.
  - The Vogel (quadratic) *IPR* correlation can be derived using the assumption of a linear mobility profile (analogous to the p^2 "deliverability" equation proposed by Fetkovich (1973)).

SPE 110821

Inflow Performance Relationship (IPR) for Solution Gas-Drive Reservoirs — Analytical Considerations

End of Presentation

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