PRESSURE BUILD-UP IN WELLS

BY

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Synopsis

The report presents a method of analysis of the pressure build-up curve obtained from a closed-in well by plotting the bottom hole pressure against the logarithm of $\frac{t_0 + \vartheta}{\vartheta}$, where ϑ is the closed-in time and t_0 is the past producing life of the well.

In all cases it proves to be possible to determine the permeability of the formation from the slope of this curve.

Mehods are also given for extrapolating the recorded pressures to infinite closed-in time for the cases of

I—a new well far from any reservoir boundary
II—a new well close to a fault, but far from any other boundary
III—a well in a finite reservoir.

Several illustrative examples are discussed.

Résumé

Le rapport présente une méthode d'analyse de la courbe de remontée de pression dans un puits fermé obtenue en représentant la pression de fond en fonction du logarithme de $\frac{t_0 + \vartheta}{\vartheta}$, ϑ étant le temps écoulé après la fermeture et t_0 la durée de la vie productive passée du puits.

Il en ressort qu'il est possible dans tous les cas de déterminer la perméabilité de la formation à partir de l'inclinaison de cette courbe.

On indique aussi des méthodes pour l'extrapolation des pressions enregistrées pour une période de fermeture infinie dans les cas ci-après:

I—un nouveau puits éloigné des limites du réservoir;
II—un nouveau puits se trouvant à proximité d'une faille, mais loin de toute autre limite;
III—un puits situé dans un réservoir limité.

Plusieurs exemples donnés à titre d'illustration sont discutés.

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Contents

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PART I—THEORY

1—Basic Equations and Assumptions

The mathematical study of the sub-surface flow of reservoir fluid requires that certain simplifying assumptions be made as to the nature of the porous medium and the fluids which it contains. In effect, the only practicable method at present available requires such sweeping simplifications in order to obtain a solution at all, that the solution so obtained requires considerable testing in practice in order to determine its usefulness and its limitations.

Thus it is general practice to develop the flow equations assuming the reservoir to be homogeneous, horizontal and of uniform thickness throughout. The fluid is assumed to obey d'Arcy's law and to be present in one phase only. Furthermore, it is assumed that the compressibility and the absolute viscosity of the fluid remain sensibly constant over the range of temperature and pressure variation encountered in the formation and that the density of the fluid obeys an exponential type law

$$\rho = \rho_0 e^{-c(p_0-p)} \quad \ldots\ldots\ldots\ldots \text{(I)}$$

where ρ is the density at some pressure p,
ρ_0 is the density at some standard pressure (conveniently taken as the original reservoir pressure p_0)
and c is the compressibility (assumed constant).

Then if we consider one well drilled into such a reservoir and assume furthermore that the flow to the well is radial (which implies either an infinite reservoir or a finite circular reservoir with the well at its centre) it may be shown that the equation of flow (1) * is

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{fc\mu}{k}\frac{\partial p}{\partial t} \quad \ldots\ldots\ldots \text{(II)}$$

where r is the distance from the centre line of the well in centimetres,
t is the time in seconds,
p is the reservoir pressure in atmospheres at distance r and time t,
f is the formation porosity expressed as a fraction of the bulk volume,
k is the formation permeability in darcies,
μ is the fluid viscosity in centipoises
and c is the fluid compressibility measured in volumes per volume per atmosphere.

Of these basic assumptions it would seem that by far the most critical is the one which requires the presence of only a single phase of reservoir fluid, in that both the compressibility and the permeability are very sensitive to changes in pressure below the bubble point. However, although the theory is developed for the case where pressures are everywhere above the bubble point, the equations often seem to fit when this condition does not hold.

* References given at end of paper.

2—Basic Solution to the Equation of Flow

There are a number of exact solutions of equation II (for various boundary conditions) given in the literature, but these suffer from the disadvantage that they involve complicated integrals and Bessel functions which make them very unwieldy for calculation purposes. We therefore make use of the so-called "point-source" solution

$$p = p_0 + \frac{q\mu}{4\pi kh} \mathrm{Ei}\left(-\frac{r^2 f\mu c}{4kt}\right)^{\dagger} \quad \ldots \text{(III)}$$

where p_0 is the initial reservoir pressure in atmospheres,
h is the formation thickness in centimetres
and q is a *constant* rate of production of the well expressed as cubic centimetres of sub-surface volume (under original conditions) per second.

This equation III is an exact solution of equation II for the following boundary conditions:
(i) external boundary at infinity at constant pressure p_0
and (ii) internal boundary (i.e. the well radius) vanishing and with a constant flow rate q across it (i.e. for a mathematical sink in an infinite reservoir).

Thus the only error introduced by using equation III as the basic solution for the case of an infinite reservoir is in considering the well radius as infinitely small. This error is considered to be negligible for the applications of this report.

3—Pressure Build-up in a Single Well in an Infinite Reservoir

a) Constant Production Rate before Closing in

Consider a single well in an infinite reservoir

† Values of the Ei-function, defined by the equation

$$\mathrm{Ei}(-x) = -\int_x^{\infty} \frac{e^{-u}}{u}\, du$$

are available from references (2) and (3).
This function is $-\infty$ for x zero and increases monotonically to zero as x goes from zero to $+\infty$. As x approaches zero, $\mathrm{Ei}(-x) - \ln x \rightarrow .5772\ldots$, and so for small values of x, say for x smaller than about .01, we may write with close approximation

$$\mathrm{Ei}(-x) \approx \ln x + .5772\ldots.$$

which was completed and first brought into production at time zero, and which subsequently produced at a constant rate q until time t_0, when it was closed in. Then, ignoring the effects of the after-production *, the well pressure p_w at time $t_0 + \vartheta$ (i.e. ϑ after closing in) may be obtained by superimposing two solutions of the form of equation III and then writing r_w for r thus

$$p_w = p_o + \frac{q\mu}{4\pi k h}\left\{ Ei\left(-\frac{r_w^2 f \mu c}{4k(t_o+\vartheta)}\right) - Ei\left(-\frac{r_w^2 f\mu c}{4k\vartheta}\right)\right\} \quad \ldots\ldots\ldots (IV)$$

where p_w = pressure at well bore in atmospheres and r_w = well radius in centimetres.

Now for small values of its argument the Ei-function may be accurately approximated by a logarithmic function. (See footnote to page 2.) If this approximation be in fact made in equation IV above we derive the basic build-up equation for a single well in an infinite reservoir as

$$p_w = p_o - \frac{q\mu}{4\pi k h} \ln \frac{t_o+\vartheta}{\vartheta} \quad \ldots\ldots (V)$$

The error involved in this approximation will be very small after a comparatively short time. It will, indeed, have dropped to ¼ % as soon as $\vartheta > \frac{25 r_w^2 f\mu c}{k}$, a condition which will usually be satisfied within a matter of seconds after closing in the well. As we have introduced errors into the calculations (by completely ignoring the after-production) which may affect the validity of our equations for a period of an hour or more after closing the well in, it is clear that both the approximation to the exact solution by the Ei-function and the approximation to the Ei-function by the ln-function are entirely justifiable.

Thus in the case of a well which has produced uniformly since completion at a rate q from an infinite reservoir we may expect the bottom hole pressure to build up in accordance with equation V.

b) Variable Production Rate before Closing in

In the previous paragraph we derived a build-up equation V for a well which produces uniformly at rate q from time zero to time t_0 and is then closed in. However, such conditions do not normally obtain, and so some correction must be applied to take account of the varying rates at which a well will have produced during its history. Two methods of correction may be used, one of which may be said to enable a theoretically precise solution to be obtained (at least in principle) while the other is nothing but a good working approximation.

To illustrate the precise method we suppose that the production history of the well was as shown by the broken line in figure 1. To this we approximate by a series of steps (as shown) and then modify the equation V to read

$$p_w = p_o - \frac{\mu}{4\pi k h}\left\{ q_o \ln \frac{t_0+\vartheta}{t_0+\vartheta-t_1} + q_1 \ln \frac{t_0+\vartheta-t_1}{t_0+\vartheta-t_2} + q_2 \ln \frac{t_0+\vartheta-t_2}{t_0+\vartheta-t_3} + q_3 \ln \frac{t_0+\vartheta-t_3}{\vartheta}\right\} \quad \ldots\ldots\ldots (VI)$$

where the t's and the q's are as indicated in fig. 1, and are so chosen that they represent the same total production as the well actually made.

However, this equation VI is rather laborious in

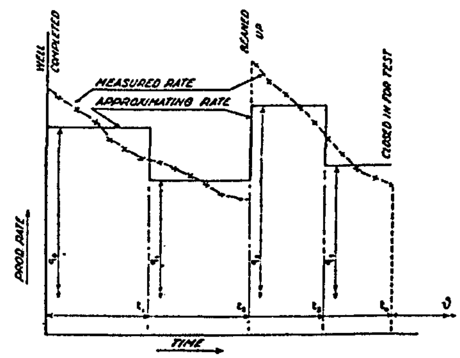


Fig. 1. Illustration of accurate method for correcting for variable production rate.

application, and will normally be more precise than is warranted by the other inaccuracies which are unavoidably present—in fact only rarely has it ever proved of value to apply this elaborate method. Instead, the equation V is usually modified by simply introducing a so-called corrected time t_c, and writing

$$p_w = p_o - \frac{q\mu}{4\pi k h} \ln \frac{t_c+\vartheta}{\vartheta} \quad \ldots\ldots (VII)$$

where q is calculated from the last established production rate before closing in the well; t_c is obtained by dividing the total cumulative production of the well by the last established production rate.

* When a well is closed in at the surface, production from the formation does not cease immediately. Instead, there is some as yet undetermined period of time during which the formation produces fluid into the well bore (at a decreasing rate) thereby building up the pressure within the well. It is this quantity of fluid—the volume which is produced by the formation into the well after closing in at the surface—which is herein termed the "after-production".

c) Interpretation of Build-up Equations V, VI, VII

It is immediately evident that if we plot p_w against $\ln\frac{t_0+\vartheta}{\vartheta}$ (using t_e for t_0 as necessary) or against $q_0 \ln\frac{t_0+\vartheta}{t_0+\vartheta-t_1} + q_1 \ln\frac{t_0+\vartheta-t_1}{t_0+\vartheta-t_2}$ + etc. if we are using the accurate correction method, we may expect the points to fall on a straight line, at least after the effects of the after-production have disappeared. If indeed it does prove possible to draw such a straight line, two deductions can be made immediately. They are:

(i) by extrapolating the line to

$$\ln\frac{t_0+\vartheta}{\vartheta} = 0$$

(or $q_0 \ln\frac{t_0+\vartheta}{t_0+\vartheta-t_1} + q_1 \ln\frac{t_0+\vartheta-t_1}{t_0+\vartheta-t_2} + \ldots$ etc. $= 0$ in the accurate case), which is equivalent to extrapolating to ϑ infinite, we may read $p_w = p_o$ which is of course equal to p, the fully built up pressure of the well. This value is, of course, identical with the initial pressure, as we are at present only considering an infinite reservoir.

(ii) The gradient of this line is equal to $\frac{q\mu}{4\pi kh}$ (or to $\frac{\mu}{4\pi kh}$ in the accurate case) and so knowing values for q, μ and h it is possible to determine a value for the permeability k measured *in situ*. This value of k has the advantage that it is a mean value for the whole well drainage area.

d) Conditions of Applicability

The theory detailed above is, however, only applicable strictly to an infinite reservoir, which is a theoretical conception which does not exist in fact. Thus the above method can only be expected to be applicable in the case of a well which has not yet produced sufficient fluid materially to have diminished the overall static reservoir pressure, i.e. a new well in which the effects of the reservoir boundary have not yet become apparent.

4—Influence of a Fault in an Otherwise Infinite Reservoir

The problem of a well producing from a point distant "a" cm from a linear barrier fault may be simply solved by the method of images. This means that instead of considering one well Q producing from a semi-infinite reservoir bounded by the linear fault (fig. 2a) we may consider two similar wells Q and Q' producing from an infinite reservoir (fig. 2b), where the fault has now been removed and the well Q' is inserted in such a manner as to have an effect equivalent to that of the fault. This requirement is simply that Q' be identical to Q and at the mirror image of Q in the fault plane, i.e. at a distance 2a from Q. Thus the pressure in Q at time $t_0 + \vartheta$ (ϑ after closing in) is

$$p_w = p_o + \frac{q\mu}{4\pi kh}\left\{ \mathrm{Ei}\left(-\frac{r_w^2 f\mu c}{4k(t_0+\vartheta)}\right) - \mathrm{Ei}\left(-\frac{r_w^2 f\mu c}{4k\vartheta}\right) + \mathrm{Ei}\left(-\frac{a^2 f\mu c}{k(t_0+\vartheta)}\right) - \mathrm{Ei}\left(-\frac{a^2 f\mu c}{k\vartheta}\right)\right\} \quad \text{(VIII)}$$

where the first two Ei-functions, unchanged from equation IV, represent the effect of the well Q, and the last two Ei-functions are the contribution of the image well Q'. As before we may substitute the ln-function for the Ei-function in the first two terms to give the build-up equation

$$p_w = p_o - \frac{q\mu}{4\pi kh}\left\{ \ln\frac{t_0+\vartheta}{\vartheta} - \mathrm{Ei}\left(-\frac{a^2 f\mu c}{k(t_0+\vartheta)}\right) + \mathrm{Ei}\left(-\frac{a^2 f\mu c}{k\vartheta}\right)\right\} \quad \text{(IX)}$$

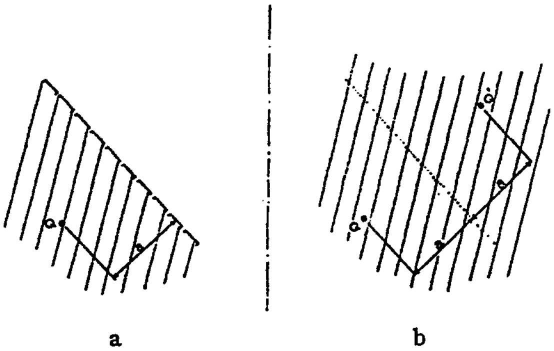


Fig. 2. Application of the method of images to the case of the linear barrier fault.

Now the distance "a" will normally be large enough to make the last term sensibly zero and the other Ei-function sensibly constant until ϑ becomes quite large. This means that for the first part of the build-up the curve will be approximately

$$p_w = p_o - \frac{q\mu}{4\pi kh}\left\{ \ln\frac{t_0+\vartheta}{\vartheta} - \mathrm{Ei}\left(-\frac{a^2 f\mu c}{k t_0}\right)\right\} \ldots \text{(X)}$$

that is the first part is a straight line of normal

$\left(\frac{q\mu}{4\pi kh}\right)$ slope when plotted against $\ln \frac{t_0+\vartheta}{\vartheta}$.

Extrapolation of this line, however, would give a final pressure below the true value (p_0). However, as ϑ becomes very large the arguments of both the Ei-functions in equation IX become small and they may then be approximated to by ln-functions. Thus for very large ϑ equation IX becomes

$$p_w = p_0 - \frac{q\mu}{2\pi kh} \ln \frac{t_0+\vartheta}{\vartheta} \quad \ldots\ldots\text{(XI)}$$

that is the last part of the build-up curve is a straight line of slope $\frac{q\mu}{2\pi kh}$ (i.e. twice the normal value) when plotted against $\ln \frac{t_0+\vartheta}{\vartheta}$; this part of the curve may be correctly extrapolated to $p_w = p_0$ at $\ln \frac{t_0+\vartheta}{\vartheta} = 0$.

Clearly there will be a range of transition values of $\ln \frac{t_0+\vartheta}{\vartheta}$ where the line of the form of equation X merges into the line of the form of equation XI. It is also theoretically possible to determine the quantity $\frac{a^2 f \mu c}{k}$ (and hence a) by fitting the build-up curve to the exact equation IX and thus to determine the distance of a fault from a well (although not, of course, its orientation).

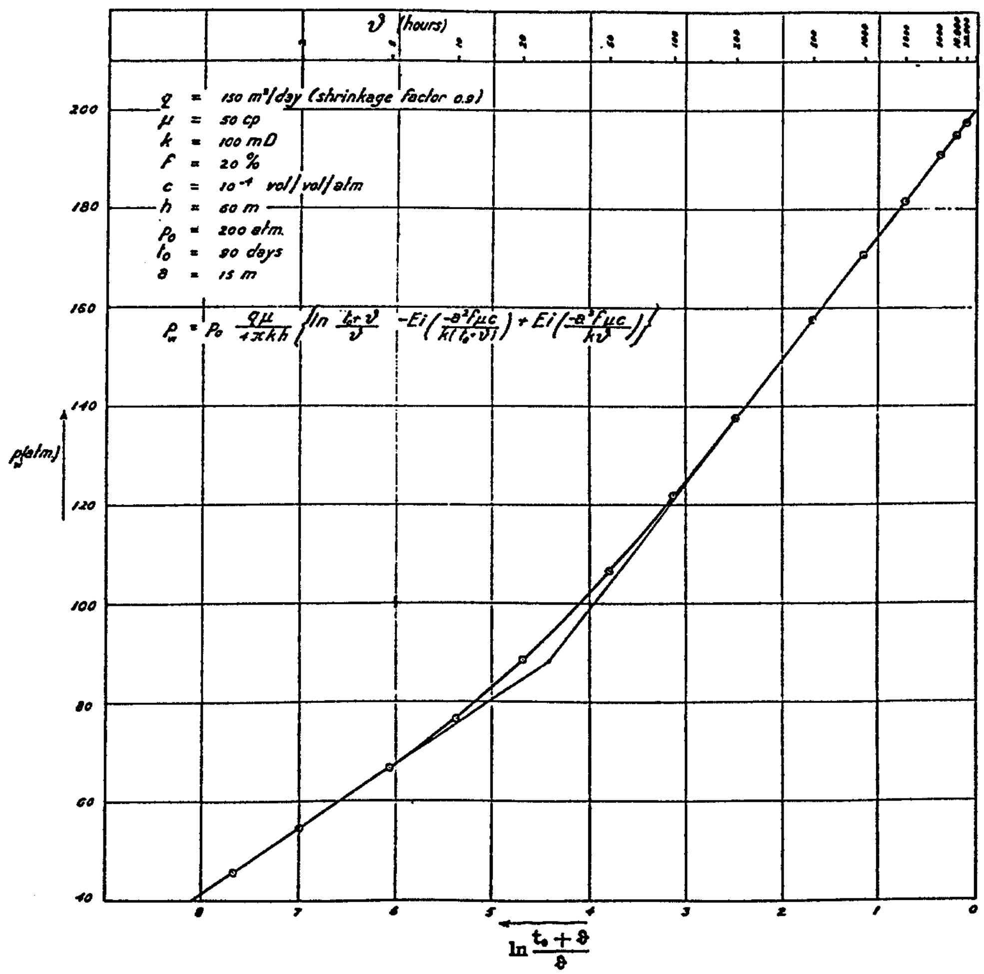


Fig. 3. Illustration of the theoretical case of a linear barrier fault.

A calculated plot of equation IX is included (fig. 3) to illustrate the form of the build-up which can be expected.

5—Well at the Centre of a Finite Circular Reservoir

Exact solutions to the flow equation II are available in the literature for the case of a single well in the centre of a finite circular reservoir. A solution (4) which only errs insofar as it treats the well as a mathematical sink in a finite reservoir (i.e. is as accurate in this case as is the Ei-solution for an infinite reservoir) is

$$p_w = p_o + \frac{q\mu}{2\pi kh}\left\{\frac{3}{4} + \ln\frac{r_w}{r_b} - \frac{r_w^2}{2r_b^2} - \frac{2kt}{f\mu c r_b^2} + 2\sum_{n=1}^{\infty}\frac{J_o\left(\frac{x_n r_w}{r_b}\right)e^{-\frac{x_n^2 kt}{f\mu c r_b^2}}}{x_n^2 J_o^2(x_n)}\right\} \quad \text{.......(XII)}$$

[where r_b = external reservoir radius (cms) and the x_n ($n = 1, 2, \ldots \infty$) are the roots of $J_1(x) = 0$], but this solution is clearly too complicated to be of other than theoretical interest.

Accordingly an approximate solution is derived by modifying the equation III. Taking r_b to be the radius of the finite reservoir boundary, we may say that the pressure drop caused by a well in an infinite reservoir is less than the pressure drop caused by an identical well in a finite reservoir by an amount dependent on the quantity of fluid which (in the infinite case) has flowed in across a circle of radius r_b. The method then is to calculate the volume of fluid which in the infinite case has crossed the external boundary r_b at any time t. Then we may say that this quantity of fluid (say Q_b), had it been produced from the finite reservoir, would have caused an additional average pressure drop throughout the finite reservoir equal to

$$\frac{Q_b}{\text{hydrocarbon-filled reservoir pore volume}} \times \frac{1}{\text{compressibility}}$$

and this quantity may be calculated to be

$$\frac{q\mu}{4\pi kh}\left\{Ei\left(-\frac{r_b^2 f\mu c}{4kt}\right) + \frac{4kt}{r_b^2 f\mu c}e^{-\frac{r_b^2 f\mu c}{4kt}}\right\}$$

Thus the pressure history of a circular field with one uniformly producing well at its centre may be obtained by including this correction term in equation III to give

$$p = p_0 + \frac{q\mu}{4\pi kh}\left\{Ei\left(-\frac{r^2 f\mu c}{4kt}\right) - y\left(\frac{r_b^2 f\mu c}{4kt}\right)\right\} \quad \text{.........(XIII)}$$

where for simplicity we define the function y by

$$y(u) = Ei(-u) + \frac{1}{u}e^{-u}$$

It is of importance to be able to consider this function y as a known function. Accordingly a plot of y for a large range in values of u is appended (fig. 10).

Now this equation XIII is not even a mathematical solution of the basic flow equation II. However, the closeness of the approximation of this equation to equation XII may be easily shown to be usually satisfactory, the error is certainly never more than a few percent. As an example fig. 4 shows the accuracy of equation XIII by comparison with equation XII in the particular case where $r_b/r_w = 1000$.

Just as was done in paragraph I, 3a, we superimpose two solutions of the form of equation XIII and write r_w for r to give our approximation to the build-up equation in a finite reservoir as

$$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} + y\left(\frac{r_b^2 f\mu c}{4k(t_0+\vartheta)}\right) - y\left(\frac{r_b^2 f\mu c}{4k\vartheta}\right)\right\} \quad \text{.........(XIV)}$$

Now until ϑ is very large the second y-function will be almost zero and the first will be nearly constant, and the equation XIV reduces to

$$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} + y(u_1)\right\} \quad \text{.....(XV)}$$

where for convenience we write u_1 for $\frac{r_b^2 f\mu c}{4kt_o}$.

Thus we still have that, over the range of values of ϑ which will normally be measured, a plot of p_w against $\ln\frac{t_o+\vartheta}{\vartheta}$ will be linear and its slope will still be $\frac{q\mu}{4\pi kh}$. However, linear extrapolation to $\ln\frac{t_o+\vartheta}{\vartheta} = 0$ will indicate a false value for the final build up (which false value we may indicate by p*) defined by

$$p^* = p_o - \frac{q\mu}{4\pi kh} y(u_1) \quad \ldots\text{.(XVI)}$$

Thus, knowing p_o and having determined $\frac{q\mu}{4\pi kh}$ and p^* from the build-up curve, we may solve equation XVI for $y(u_1)$ and thus we may derive a value of $u_1 = \frac{r_b^2 f \mu c}{4 k t_o}$.

Now if we return to the former equation XIV and let ϑ become infinite, the equation becomes

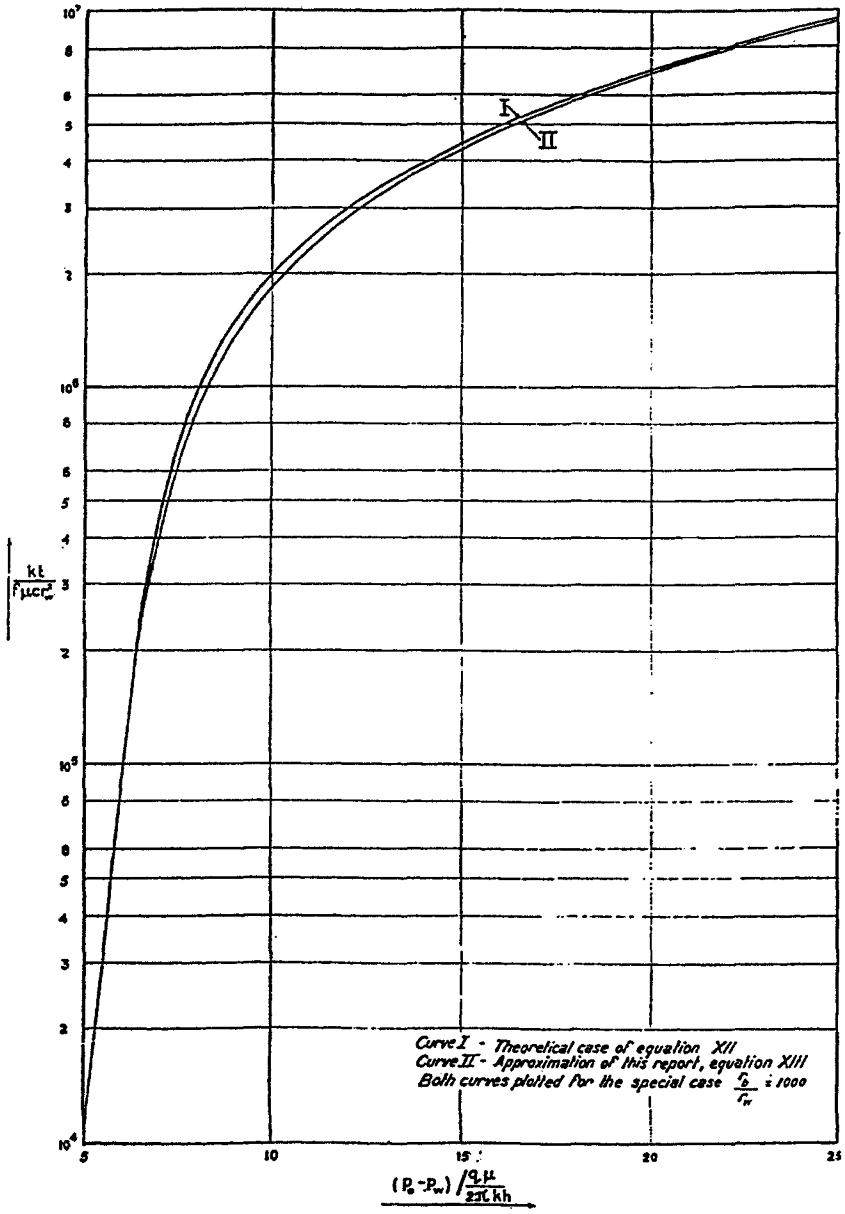


Fig. 4. Comparison between precise theory and approximation of this report for the case of a well in a finite circular reservoir.

$$p_s = p_o - \frac{q t_o}{\pi r_b^2 f h c} = p_o - \left(\frac{q\mu}{4\pi k h}\right) \div \left(\frac{r_b^2 f \mu c}{4 k t_o}\right)^{\dagger} \quad \ldots\ldots\ldots \text{(XVII)}$$

where p is the final static (i.e. fully built up) closed in pressure, and then, substituting the known values of p_o, $\frac{q\mu}{4\pi kh}$ and $\frac{r_b^2 f\mu c}{4kt_o}$ in XVII we may obtain a value of p_s, thereby correctly extrapolating the build-up curve.

PART II—APPLICATIONS

1—Introduction

Part II of this report is intended to be largely complete in itself. The object of this part is, by the use of several examples, to illustrate the methods of applying the more important equations developed in Part I. These equations are collected for easy reference in the Summary of Equations Chart, Appendix A; each equation is repeated in two forms namely exactly as derived in Part I and also as modified for use with practical oil-field units. All the symbols used in the report are collected in a table (Appendix B) to which reference should be made for the units which are to be used for the various quantities involved, and for the values of the numerical constants of conversion (A, B, D, F and G) to be used with the particular units employed.

The examples chosen to illustrate the use of the methods here presented are all from wells in the Casabe field, Colombia, and have been selected from a batch of 66 pressure surveys in this field made between August, 1948, and April, 1950. Those which have been selected for inclusion in this report have been so chosen because they clearly emphasize certain points of particular interest; it must be emphasized that the accuracy and compatibility of the experimental data obtained from the five wells here considered is in no way superior to that of the other 61 which are not included.

It is perhaps of interest to state that the oilbearing formations of the Casabe field consist of three series of lenticular multiple sands 'A', 'B' and 'C' of which the 'A' and 'B' sands are of Oligocene and the 'C' sand is of Eocene age. They are fine grained to silty, rather argillaceous, and fairly well consolidated. The oil is heavy (20°-21° API) and viscous (40 cp ±). Some doubt exists as to the bubble point. It was first thought that the oil in all sands was undersaturated, but it now appears that pressures in the 'A' sands at least are below the bubble point. This, however, does not appear to affect the build-up curves.

2—Examples of the Pressure Build-up in a New Well

The earlier theory developed in Part I (I, 3) is designed to apply to a single well in an infinite field. Such a condition never obtains, of course, but it is further pointed out (I, 3d) that the case of a *new* well in a finite field is similar so long as the total withdrawal from the well has been kept small. It is difficult to lay down a criterion for the order of this smallness—practical applications seem to indicate that a normal well may be allowed to produce for a period of weeks or even of several months and it will still obey the "infinite reservoir" theory. Thus we may apply the infinite reservoir theory to the following case.

CB-161 was completed to the 'A' sands on 7th February 1950, and it was closed in from 16th February to 8th March while continual bottom hole pressure measurements were being taken. As will be apparent from the analysis of the results, it was not really necessary to leave the well closed in for such a long period in order to obtain adequate data, although it is interesting to observe how closely the pressure in the well followed the predicted course over such a long time.

This being a new well, we apply the theory for a single well in an infinite reservoir, as has been already explained. Reference to the Summary of Equations Chart, Appendix A, shows that the equation for the build-up in this case is number V, which in practical units is

$$p_w = p_o - \frac{G q \mu}{C_o k h} \log_{10} \frac{t_o + \vartheta}{\vartheta} \quad \ldots\ldots \text{(Va)}$$

Reference to the table of Appendix B gives a definition of these symbols and of the units in which they are to be measured. The U.S. system of units being in use in Casabe, we must use the value of G given in the tabulation as pertaining to this system, namely 162.6. Thus equation Va is modified to

$$p_w = p_o - \frac{162.6\, q \mu}{C_o k h} \log_{10} \frac{t_o + \vartheta}{\vartheta} \quad \ldots \text{(Vb)}$$

which is thus the equation which we must attempt

† It is of interest to observe that this limiting form of equation XIV may be expressed thus (Present Reservoir Pressure) = (Initial Pressure) — $\frac{\text{(Total Volume Withdrawn)}}{\text{(Total Reservoir Pore Volume)}} \times \frac{1}{\text{Compressibility}}$ which is precisely what we should expect from elementary considerations.

to fit to the experimental data. As the method of analysis consists of plotting $\log_{10} \frac{t_e + \vartheta}{\vartheta}$ against p_w, we firstly require a value for t_0. As has been previously explained (I, 3b) the equation V (or its modified form, Vb) assumes that q, the rate of production, has remained constant for the whole life of the well (t_0). This not being in fact true, the method of correction is to take the last available production rate—in this particular case, 641 bbl/day—as the value for q, and to use a corrected time t_e, instead of the theoretical t_0, where t_e is obtained by dividing the total cumulative production of the well (in this case, 5847 bbl) by this last production rate.

Hence for t_e in Vb above we substitute

$$t_e = \frac{\text{Total cum. prod.}}{\text{Last prod. rate}} = \frac{5847}{641} \text{ days} = 219 \text{ hours}$$

Thus the build-up equation Vb may be modified by the insertion of this value of t_e for t_o. This gives

$$p_w = p_o - 162.6 \frac{q\,\mu}{C_o\,k\,h} \log_{10} \frac{219 + \vartheta}{\vartheta} \;.. \text{(Vc)}$$

The value of q = 641 bbl/day could now be inserted, but it is perhaps more convenient to leave this until a later stage.

The method of treating the experimentally obtained pressure data may be most easily explained by reproducing the measured values exactly as they were reported from well CB-161.

Pressure p_w (psig)	C.I. time ϑ (hours)	$\frac{219 + \vartheta}{\vartheta}$	$\log_{10} \frac{219 + \vartheta}{\vartheta}$
1192	19	12.53	1.0980
1200	25	9.760	.9894
1206	31	8.065	.9066
1212	37	6.919	.8400
1216	43	6.093	.7848
1220	49	5.469	.7379
1223	55	4.982	.6974
1227	61	4.590	.6618
1230	67	4.269	.6303
1232	73	4.000	.6021
1235	79	3.772	.5766
1236	85	3.576	.5534
1237	91	3.407	.5324
1239	97	3.258	.5130
1241	103	3.126	.4950
1242	109	3.009	.4784
1241	115	2.904	.4630
1243	121	2.810	.4487
1244	127	2.724	.4352
1245	133	2.647	.4228
1247	139	2.576	.4109
1249	145	2.510	.3997
1249	151	2.450	.3892
1250	157	2.395	.3793
1267	477	1.459	.1641

Columns 1 and 2 are derived directly from the bottom hole pressure readings, columns 3 and 4 are calculated from column 2, and finally the values of p_w from column 1 are plotted against the corresponding values of $\log_{10} \frac{219 + \vartheta}{\vartheta}$ from column 4 as in figure 5.

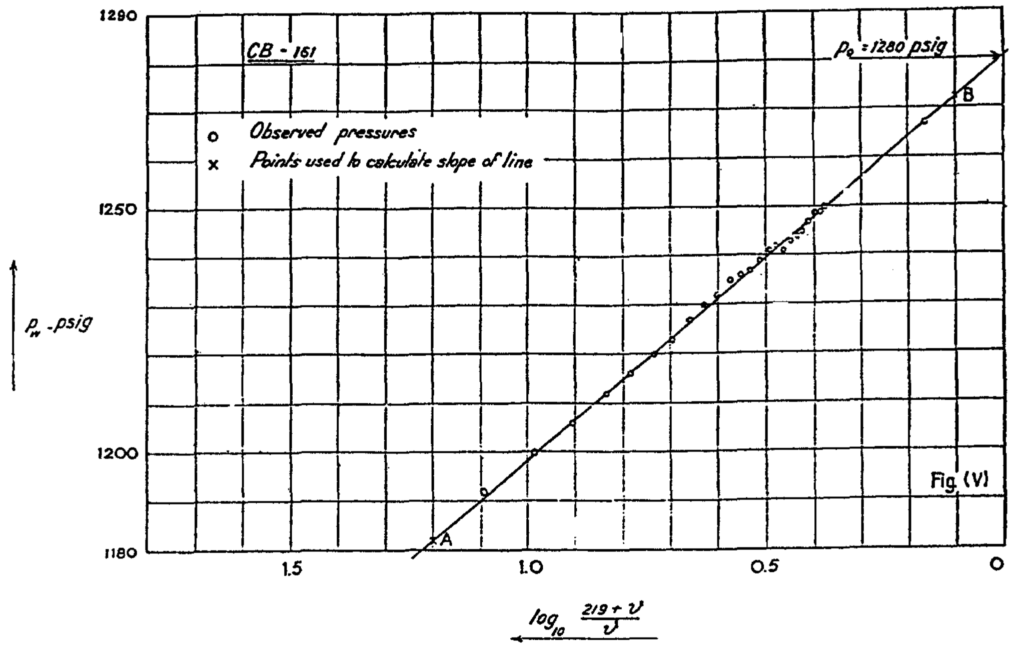
It is convenient to plot p_w vertically and in a conventional manner, but to plot $\log_{10} \frac{219 + \vartheta}{\vartheta}$ horizontally from right to left, i.e. with the zero on the right hand side as has been done in fig. 5, as this gives a more vivid impression of rising pressure.

If agreement is to be obtained with the theory, and thus with the derived equation Vc, the points when so plotted should fall on a straight line, excepting only possibly the points corresponding to small ϑ when the effects of the after-production (see footnote on page 3) may be still felt.

As can be seen from fig. 5, the accuracy with which these experimental points do in fact plot on a straight line in this case—and particularly the very last point which represents nearly 20 days closed in—is really quite remarkable.

The interpretation of this figure 5 is simple. Firstly we may deduce a value for p_0, which is done by extending the straight line plot to the point corresponding to $\log_{10} \frac{219 + \vartheta}{\vartheta} = 0$ and reading the corresponding pressure which gives in this case a value for p_0 of 1280 psig. Although we have defined p_0 to be the initial reservoir pressure, this must be interpreted somewhat widely. In this case, for example, the well considered was drilled as an infilling well into an already heavily drilled field, and so the pressure p_0 = 1280 psig must be interpreted as the reservoir pressure at the well *at the moment of its completion.*

In addition to this pressure determination we may determine the permeability k from the slope of the straight line. This can perhaps best be done by selecting two arbitrary and fairly widely separated points A and B on the line (fig. 5). The pressure difference between A and B is 1272—1182 = 90 psi, and the corresponding difference in $\log_{10} \frac{219 + \vartheta}{\vartheta}$ is 1.2 — .1 = 1.1. The slope of the line is then obtained by division thus



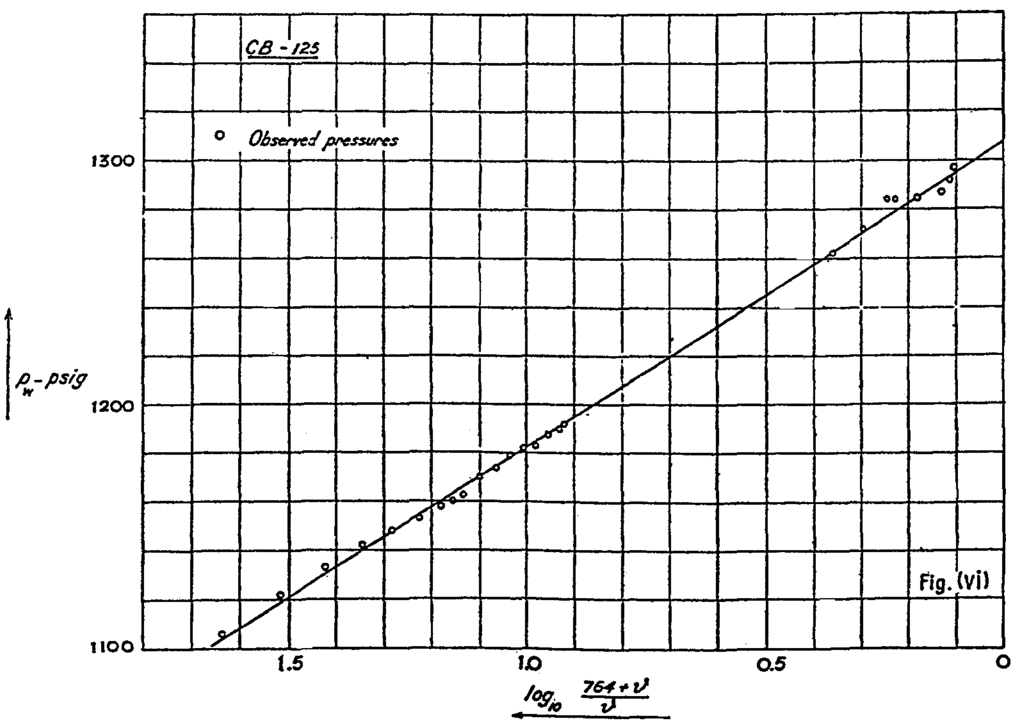


Fig. 5 and 6. Observed pressure build-up curves in new wells.

$$\text{Slope} = \frac{\text{Difference in pressure}}{\text{Difference in } \log_{10}\frac{219+\vartheta}{\vartheta}}$$

$$= \frac{90}{1.1} = 82$$

and this slope we now equate to the coefficient of $\log_{10}\frac{219+\vartheta}{\vartheta}$ in the build-up equation Vc. Thus we have

$$82 = 162.6\,\frac{q\mu}{C_o k h}$$

from which we may deduce

$$\frac{C_o k h}{q\mu} = \frac{162.6}{82} = 1.98 \quad \ldots\ldots\text{(XVIII)}$$

In the particular case of the well CB-161 the following additional data are available:

Rate of production $q = 641$ bbl/day (Ref. page 9)
Viscosity $\mu = 40$ cp } From PVT data
Shrinkage factor $C_0 = .93$ }
Pay thickness $h = 349$ ft From Electric log
and if we substitute these values of μ, C_0 and h in the above equation XVIII we have

$$\frac{.93 \times k \times 349}{641 \times 40} = 1.98$$

that is $.0127\ k = 1.98$

or $k = \dfrac{1.98}{.0127} = 156\,\text{mD}$

Thus we have derived a value of 1280 psig for the reservoir pressure and 156 md for the permeability, both of which are in good agreement with other data for the Casabe field.

Figure 6 shows an example of the build-up of another new well. This requires no further comment, except to note that it is included only because it is an exceptionally good example of a very long period closed in.

The data relevant to this example are:

Well No CB-125, A sands
$q = 280$ bbl/day
$t_e = 764$ hours (31.8 days)
Max. C. I. time 2847.7 hours (118.7 days)
k (from the slope of the line) = 88 md

The points corresponding to very large closed-in times do not, of course, plot with the same accuracy on the straight line. This is because the earlier group of 17 points were obtained from one (or perhaps two) runs of the pressure gauge, while the last 8 points are eight spot readings taken at widely spaced values of ϑ.

3—Examples of the Pressure Build-up in a Well near a Linear Barrier Fault

The theory which has been developed for a linear barrier fault is strictly only applicable to a well in an otherwise infinite reservoir. However, we may approximate to this condition by a new well close to a fault and considerably farther from any other barrier. Such a well is CB-123, completed to the C-sands at the beginning of September 1949; it was closed in for test from 4th November 1949 to 5th January 1950. Reference to Appendix A shows that the relevant equations are numbers IX, X and XI, which, subsituting the values of A, D, F and G appropriate to U.S. units from Appendix B, become

$$p_w = p_o - \frac{q\mu}{C_o kh}\left\{162.6\log_{10}\frac{t_o+\vartheta}{\vartheta} - 70.60\,\text{Ei}\left(-\frac{3793\,a^2 f\mu c}{k(t_o+\vartheta)}\right) + 70.60\,\text{Ei}\left(-\frac{3793\,a^2 f\mu c}{k\vartheta}\right)\right\} \quad \ldots\text{(IXa)}$$

which is valid for all ranges of ϑ, and which may be approximated to by

$$p_w = p_o - \frac{q\mu}{C_o kh}\left\{162.6\log_{10}\frac{t_o+\vartheta}{\vartheta} - 70.60\,\text{Ei}\left(-\frac{3793\,a^2 f\mu c}{k t_o}\right)\right\} \quad \ldots\text{(Xa)}$$

for all save very large values of ϑ, and by

$$p_w = p_o - 325.1\,\frac{q\mu}{C_o kh}\log_{10}\frac{t_o+\vartheta}{\vartheta} \quad \ldots\text{(XIa)}$$

for very large ϑ.

The exact equation IXa is not of great interest as far as the applications are concerned. Instead, the two approximations Xa and XIa are used.

Firstly we require a value of t_0, which is derived as for the previous example. We use $q = 275$ bbl/day and $t_0 = 1353$ hours. The value of q, as before, we do not yet substitute. However, we insert the value of t_0 in the two approximate equations which become

$$p_w = p_o - \frac{q\mu}{C_o kh}\left\{162.6\log_{10}\frac{1353+\vartheta}{\vartheta} - 70.60\,\text{Ei}\left(-\frac{3793\,a^2 f\mu c}{1353\,k}\right)\right\} \quad \ldots\text{(Xb)}$$

for all but very large ϑ, and

$$p_w = p_o - 325.1\,\frac{q\mu}{C_o kh}\log_{10}\frac{1353+\vartheta}{\vartheta} \quad \text{(XIb)}$$

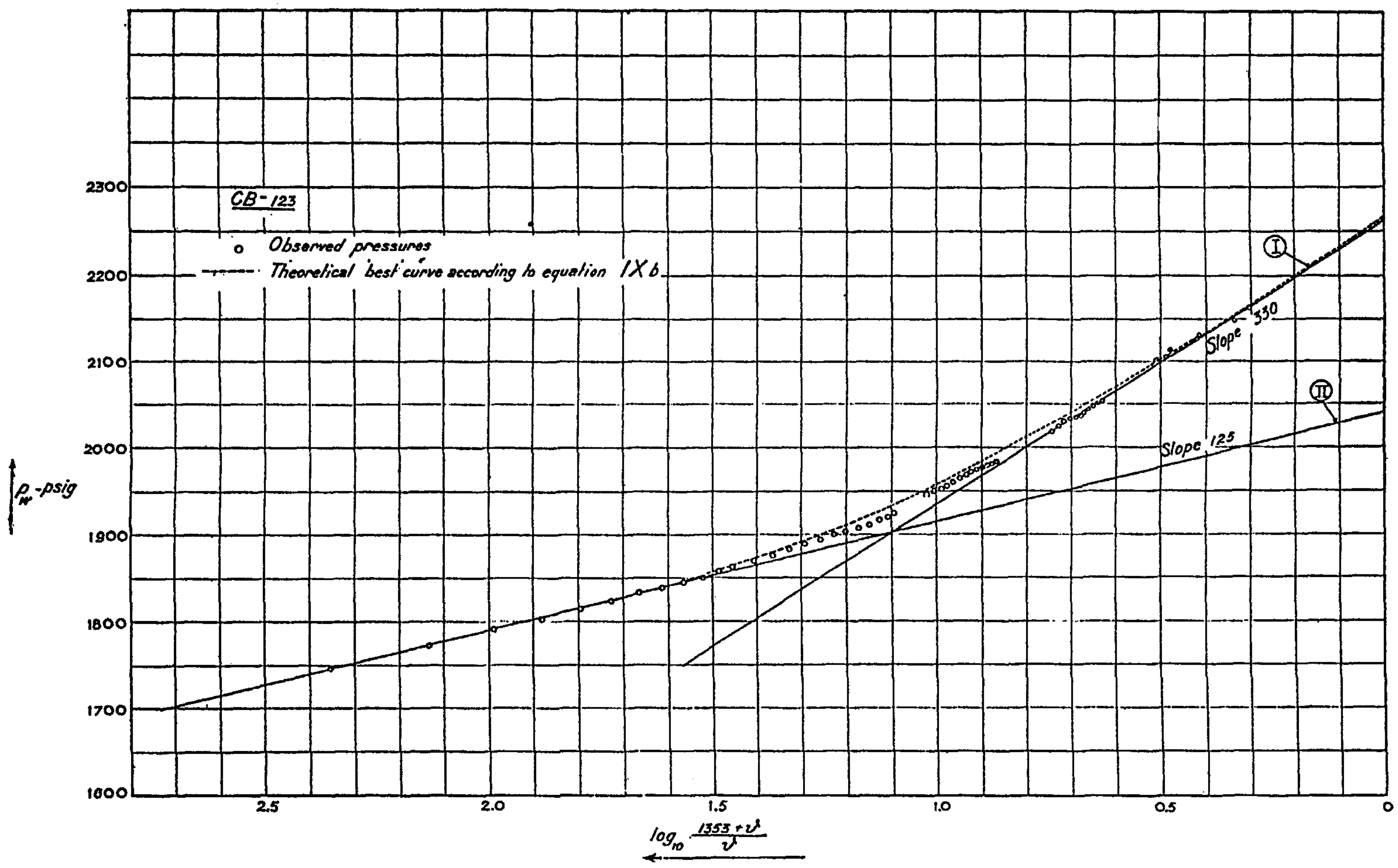


Fig. 7. Observed pressure build-up curve in well affected by a linear barrier fault.

for very large ϑ. Just as in the previous example, we plot p_w against $\log_{10} \frac{1353+\vartheta}{\vartheta}$. The resulting plot is shown in fig. 7. Theoretical requirements are that the early part of the curve be a straight line (represented by equation Xb) and that the latter part be also a straight line (according to equation XIb) of twice the slope of the early part. These two straight lines should be joined by a transition zone.

There are, indeed, two well defined straight lines I and II, but their slopes are respectively 330 and 125, i.e. in a ratio of 2.64 : 1, instead of 2 : 1. This may be explained, however, by the fact that the particular well in question is subjected to the influence of two intersecting faults, the effect of which may be theoretically shown to be to increase the ratio of the two slopes.

As before, a value of p_0 may be obtained by extrapolating the line I to give $p_0 = 2263$ psig.

The value of k is obtained from the slope of the straight line II, and proceeds just as before:

$$162.6 \frac{q\,\mu}{C_o\,k\,h} = \text{slope of line II} = 125$$

and substituting the known values

$q = 275$ bbl/day (Ref. page 11)
$\mu = 40$ cp, $C_0 = .93$ } from PVT data
$h = 57$ ft (Electric log)

we have

$$162.6 \frac{275 \times 40}{.93 \times k \times 57} = 125$$

i.e. $\frac{33{,}700}{k} = 125$

whence $k = 270$ md.

We may also estimate the distance of the fault from the well by the following method.

We equate the function $-\mathrm{Ei}\left(-\frac{3793\,a^2 f \mu c}{1353\,k}\right)$ to the value of $2.3 \log_{10} \frac{1353+\vartheta}{\vartheta}$ † at the point of intersection of the two straight lines I and II (fig. 7).

These lines intersect at $\log_{10} \frac{1353+\vartheta}{\vartheta} = 1.1$, and so we have the equation

$$-\mathrm{Ei}\left(-\frac{3793\,a^2 f \mu c}{1353\,k}\right) = 2.3 \times 1.1 = 2.53 \quad \text{..(XIX)}$$

Now if we substitute the known values

$f = .25$
$\mu = 40$ cp
$c = 5.1 \times 10^{-6}$ vols/vol/psi
$k = 270$ md

the above equation XIX becomes

$$-\mathrm{Ei}\left(-\frac{3793 \times a^2 \times .25 \times 40 \times 5.1 \times 10^{-6}}{1353 \times 270}\right) = 2.53$$

i.e. $-\mathrm{Ei}\,(-.530 \times 10^{-6}\,a^2) = 2.53$

and reference to a table of Ei-functions (2, 3) shows that, for this value of $-\mathrm{Ei}\,(-.530 \times 10^{-6}\,a^2)$ the quantity

$$.530 \times 10^{-6}\,a^2 = .0468$$

whence $a^2 = 8.83 \times 10^4$
or $a = 297$ feet

However, this figure is not in agreement with the present (rather obscure) subsurface picture which places the fault at about 1100 feet from the well.

Instead of proceeding in the manner just detailed, however, we may modify our approach thus.

Firstly, we accept the possibility that the slopes of the lines I and II (figure 7) may not be exactly in the ratio of 2 : 1. If we suppose this ratio to be b : 1 a first approximation to the build-up equation may be obtained by modifying equation IX to read (in practical units)

$$p_w = p_o - \frac{q\,\mu}{C_o\,k\,h}\left\{G \log_{10} \frac{t_o+\vartheta}{\vartheta} - (b-1)\,A\,\mathrm{Ei}\left(-\frac{D\,a^2 f \mu c}{k\,(t_0+\vartheta)}\right) + (b-1)\,A\,\mathrm{Ei}\left(-\frac{D\,a^2 f \mu c}{k\,\vartheta}\right)\right\} \quad \text{..(IXb)}$$

The problem of fitting this equation to the observed points can then be performed thus.

An approximate value of "b" is derived from the ratio of the two slopes, a value of p_0 is obtained by the extrapolation of the last part of the build-up curve and a value of "a" is obtained just as previously described i.e. by equating the $-\mathrm{Ei}$ function to $2.3 \times \log_{10} \frac{t_0+\vartheta}{\vartheta}$ at the point of intersection of the two straight lines. It is convenient that, given this point of intersection, the value of "a" thereby determined is independent of the slopes of the lines.

Due to some uncertainty in the value of "b" (for in practice ϑ must usually be very large indeed before line I is thoroughly established, and so it is possible for some of the later points in the transition zone to be mistaken for points on this line I) it may not now be taken as definite that the best possible values of a, b and p_0 have now been chosen, although it may be expected that they will not differ too widely from the final values.

Thus the equation IXb is now plotted to see how nearly the calculated curve fits the observed points;

† Note that the factor 2.3 is a universally applicable constant for this method (actually it is ln 10).

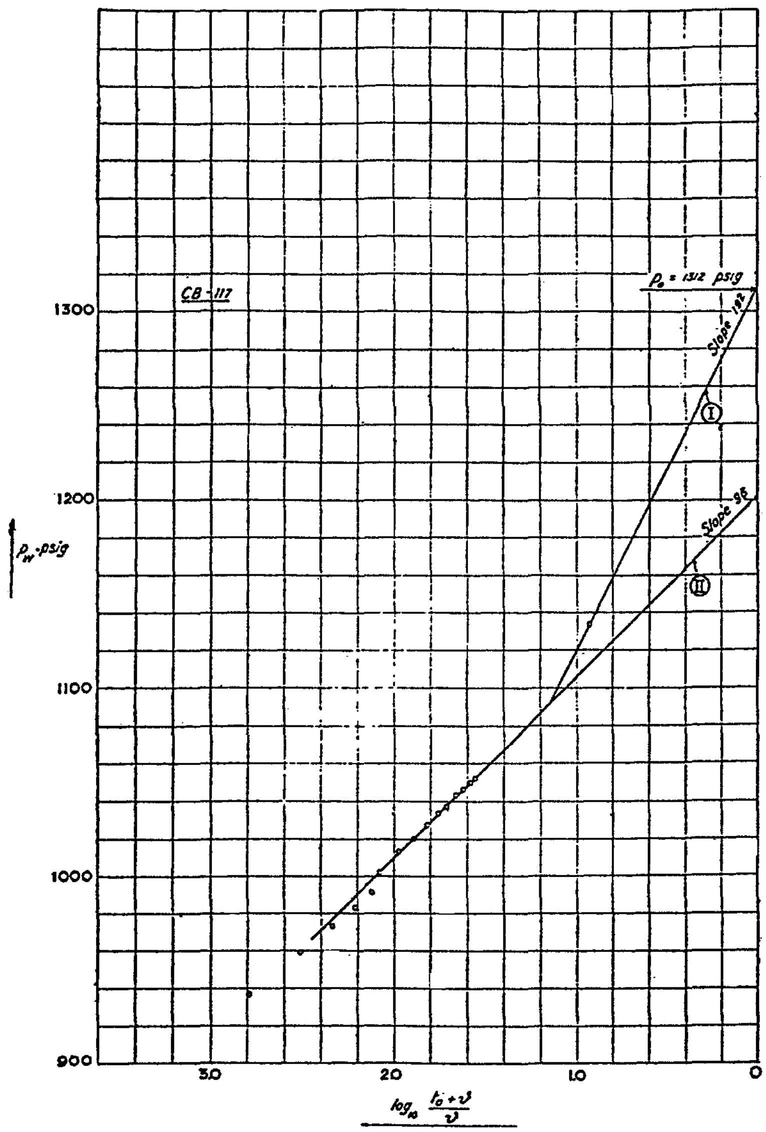


Fig. 8. Observed pressure build-up curve in well possibly affected by a linear barrier fault.

in the event of an unsatisfactory fit, the three quantities a, b and p_0 may be somewhat modified, and a process of trial and error will finally lead to the best values. Fortunately this seems to be fairly quick in application.

The end result of such a process may be seen in figure 7, in which the broken line has been calculated to fit the observed points using the method outlined above.

The final "best" values of the quantities a, b and p_0 were not widely divergent from the values already obtained, as may be seen in the following tabulation

	Best values from Eqn IXb	Previously obtained values
a	325 ft	297 ft
b	2.81	2.64
p_o	2270 psi	2263 psi

There is a further possible use of this linear barrier fault theory.

Figure 8 shows the usual p_w vs. $\log_{10}\frac{t_0+\vartheta}{\vartheta}$ plot of well CB-117. The unexpectedly high pressure reading at 1134 psig may be easily explained by the presence of a fault. Thus a straight line II may first be drawn through the other (earlier) points, and its slope measured—in this example it has a value of 96 (whence k may be obtained as before, formula Xb).

Then through the exceptionally high pressure point at 1134 psig we draw a line I at twice the slope of the line II, i.e. at a slope of 192 and extrapolate this to give a value of $p_0 = 1312$ psig.

This method must, of course, be used with considerable discretion.

4—Example of the Pressure Build-up in a Well in a Finite Reservoir

The method again is to plot p_w against $\log_{10}\frac{t_0+\vartheta}{\vartheta}$ and from the straight line plot so obtained to calculate k. However, linear extrapolation of the straight line region of this build-up curve does not give the value of the reservoir pressure.

As an example of the method we include the results of a long survey made on well CB-110 (p_w plotted against $\log_{10}\frac{t_0+\vartheta}{\vartheta}$) figure 9. The relevant equations are XV, XVI and XVII. We proceed thus:

The slope of the initial straight part is measured to be 121, and then from equation XV we equate this value to the coefficient of $\log_{10}\frac{t_0+\vartheta}{\vartheta}$ to give an equation $121 = G\frac{q\mu}{C_o k h}$ and taking G = 162.6 (U.S. units, Appendix B) we get

$$\frac{q\mu}{C_o k h} = \frac{121}{162.6} = .744$$

which may be solved for k, knowing q, μ, C_0 and h, as before.

If we extrapolate the straight line, however, we get a false pressure reading $p^* = 1313$ psig. To correct this we also need to know the value of p_0. We take in this case 1343 psig (as measured in a

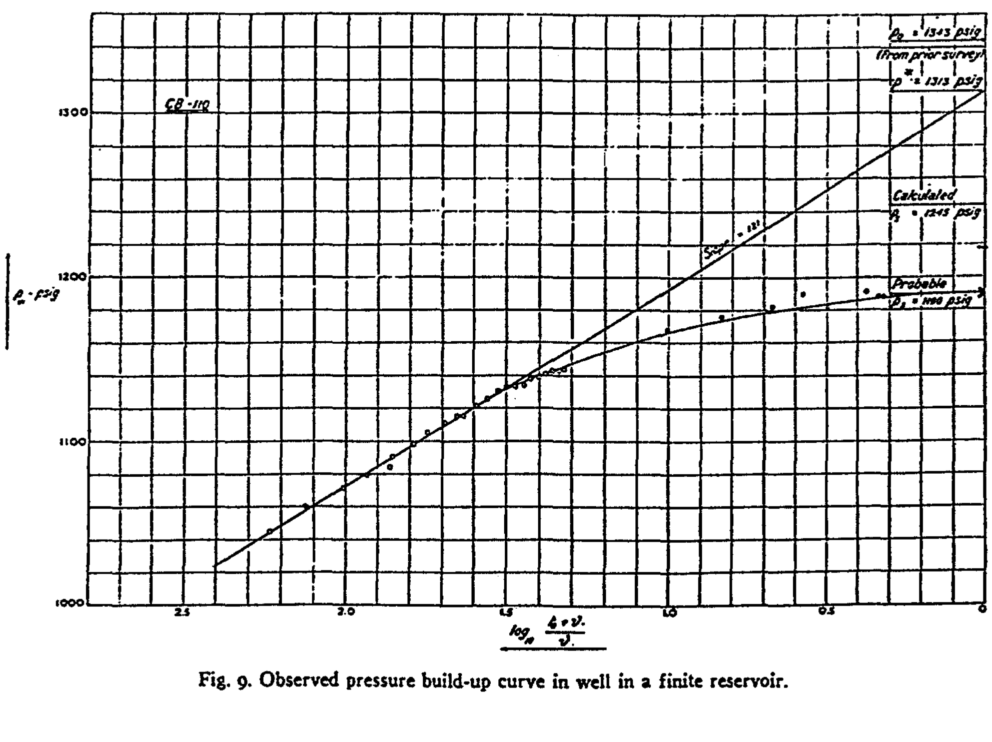


Fig. 9. Observed pressure build-up curve in well in a finite reservoir.

previous pressure survey) and use equation XVI, which is (Ref. Appendix A)

$$p^* = p_o - A\,\frac{q\mu}{C_o kh}\,y\left(\frac{Br_b^2 f\mu c}{k t_0}\right) \quad \text{(XVIa)}$$

Substituting then the values

$p^* = 1313$ psig } as above
$p_0 = 1343$ psig }
$A = 70.60$ (ref. Appendix B)

and $\frac{q\mu}{C_o kh} = .744$ (from slope of linear part of curve, as calculated above)

in equation XVIa, we have

$$1313 = 1343 - 70.60 \times .744\; y\left(\frac{Br_b^2 f\mu c}{k t_0}\right)$$

which may be solved for y to give

$$y\left(\frac{Br_b^2 f\mu c}{k t_0}\right) = \frac{1343 - 1313}{70.60 \times .744} = .571$$

This y-function is plotted in fig. 10. Reference to this figure shows that if $y(u) = .571$, then $u = .537$.

Thus we have for this case

$$\frac{Br_b^2 f\mu c}{k t_0} = .537$$

Now we go to equation XVII, which we write

$$p_s = p_o - A\,\frac{q\mu}{C_o kh} \div \left(\frac{Br_b^2 f\mu c}{k t_0}\right) \ldots\ldots \text{(XVIIa)}$$

and we substitute the values

$p_0 = 1343$ psig
$A = 70.60$
$\frac{q\mu}{C_o kh} = .744$
$\frac{Br_b^2 f\mu c}{k t_o} = .537$

as obtained above

in this equation XVIIa to give

$$p_s = 1343 - 70.60 \times .744 \div .537$$
$$= 1343 - 98$$
$$= 1245 \text{ psig}$$

as the calculated static pressure at the well after infinite time closed in.

Agreement is here not too good—the apparent error in p_s is about 55 psi as may be seen from figure 9. This is, as yet, the only well which we have had closed in for long enough to be able to make such a check. Accordingly this method must still be treated as unproven.

5—Comments

In the application of these methods there are certain definite dangers. Firstly it is not always clear which part of the build-up curve is to be used to determine k. It is not uncommon for many of the early pressure readings to fall on a straight line, when plotted against $\log_{10}\frac{t_0+\vartheta}{\vartheta}$ although they have been taken during the period of the after-production. This report does not attempt to cover this period of after-production, and this early straight line, having a slope often many times greater than the true value of $\frac{Gq\mu}{C_o kh}$ will thus give incorrect values of k and p_0.

Secondly, when the after-production has ceased, all wells show an initial slope of $\frac{Gq\mu}{C_o kh}$ and yet for extrapolation purposes it is necessary to know which case must be catered for; infinite, semi-infinite or finite reservoir. Thus a good deal of care must be exercised in analysing the results of build-up curves if reliable data are to be obtained.

After analysing a large number of wells by this method we have obtained the impression that acceptable values for the permeability are more frequently obtained than for the extrapolated pressure. This is probably due to the fact that whereas one is content with only an approximate value of k the limits of error within which the reservoir pressure is required are very much smaller.

Bibliography

(1) Muskat, Flow of Homogeneous Fluids Through Porous Media, Chap. X, Sec. 10.2, Eqn 1.

(2) Jahnke und Emde, Funktionentafeln Section 1, pp. 6-8.

(3) Tables of Sine, Cosine and Exponential Integrals, Vols I and II, Federal Work Agency, Works Project Administration for the City of New York.

(4) Muskat, Flow of Homogeneous Fluids Through Porous Media, Chap. X, Sec. 10.13, Eqn 2.

APPENDIX A—SUMMARY OF EQUATIONS CHART

EQUATION No.	REF. PAGE No.	BOUNDARY CONDITIONS	QUANTITY EVALUATED	RANGE OF VALUES OF TIME ϑ FOR WHICH EQN. IS VALID ‡	THE MORE IMPORTANT EQUATIONS	
					a) EXPRESSED IN THE UNITS OF PART I	b) MODIFIED FOR USE WITH PRACTICAL OIL-FIELD UNITS
III	2	Single well in an infinite reservoir (or a new well in a finite reservoir)	Pressure at any point	All values	$p = p_0 + \frac{q\mu}{4\pi kh} Ei\left(-\frac{r^2 f\mu c}{4kt}\right)$	$p = p_0 + \frac{Aq\mu}{C_0 kh} Ei\left(-\frac{Br^2 f\mu c}{kt}\right)$
V	3		Pressure build-up in a closed-in well	All values	$p_w = p_0 - \frac{q\mu}{4\pi kh} \ln\frac{t_0+\vartheta}{\vartheta}$	$p_w = p_0 - \frac{Gq\mu}{C_0 kh}\log_{10}\frac{t_0+\vartheta}{\vartheta}$
IX	4	Single well in a semi-infinite reservoir (i.e. a linear barrier fault in an otherwise infinite reservoir)		All values	$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} - Ei\left(-\frac{a^2 f\mu c}{k(t_0+\vartheta)}\right) + Ei\left(-\frac{a^2 f\mu c}{k\vartheta}\right)\right\}$	$p_w = p_0 - \frac{q\mu}{C_0 kh}\left\{G\log_{10}\frac{t_0+\vartheta}{\vartheta} - A.Ei\left(-\frac{Da^2 f\mu c}{k(t_0+\vartheta)}\right) + A.Ei\left(-\frac{Da^2 f\mu c}{k\vartheta}\right)\right\}$
X	4			All except very large values	$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} - Ei\left(-\frac{a^2 f\mu c}{kt_0}\right)\right\}$	$p_w = p_0 - \frac{q\mu}{C_0 kh}\left\{G\log_{10}\frac{t_0+\vartheta}{\vartheta} - A.Ei\left(-\frac{Da^2 f\mu c}{kt_0}\right)\right\}$
XI	5			Very large values	$p_w = p_0 - \frac{q\mu}{2\pi kh}\ln\frac{t_0+\vartheta}{\vartheta}$	$p_w = p_0 - \frac{Fq\mu}{C_0 kh}\log_{10}\frac{t_0+\vartheta}{\vartheta}$
XIII	6	Single well at the centre of a finite circular reservoir	Pressure at any point	All values	$p = p_0 + \frac{q\mu}{4\pi kh}\left\{Ei\left(-\frac{r^2 f\mu c}{4kt}\right) - y\left(\frac{r_b^2 f\mu c}{4kt}\right)\right\}$	$p = p_0 + \frac{Aq\mu}{C_0 kh}\left\{Ei\left(-\frac{Dr^2 f\mu c}{kt}\right) - y\left(\frac{Br_b^2 f\mu c}{kt}\right)\right\}$
XIV	6		Pressure build-up in a closed-in well.	All values	$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} + y\left(\frac{r_b^2 f\mu c}{4k(t_0+\vartheta)}\right) - y\left(\frac{r_b^2 f\mu c}{4k\vartheta}\right)\right\}$	$p_w = p_0 - \frac{q\mu}{C_0 kh}\left\{G\log_{10}\frac{t_0+\vartheta}{\vartheta} + A.y\left(\frac{Dr_b^2 f\mu c}{k(t_0+\vartheta)}\right) - A.y\left(\frac{Dr_b^2 f\mu c}{k\vartheta}\right)\right\}$
XV	6			All except very large values	$p_w = p_0 - \frac{q\mu}{4\pi kh}\left\{\ln\frac{t_0+\vartheta}{\vartheta} + y\left(\frac{r_b^2 f\mu c}{4kt_0}\right)\right\}$	$p_w = p_0 - \frac{q\mu}{C_0 kh}\left\{G\log_{10}\frac{t_0+\vartheta}{\vartheta} + A.y\left(\frac{Dr_b^2 f\mu c}{kt_0}\right)\right\}$
XVI	7		Extrapolated static closed-in pressure	—	$p^* = p_0 - \frac{q\mu}{4\pi kh} y\left(\frac{r_b^2 f\mu c}{4kt_0}\right)$	$p^* = p_0 - \frac{Aq\mu}{C_0 kh} y\left(\frac{Br_b^2 f\mu c}{kt_0}\right)$
XVII	8			ϑ infinite	$p_s = p_0 - \left(\frac{q\mu}{4\pi kh}\right) \div \left(\frac{r_b^2 f\mu c}{4kt_0}\right)$	$p_s = p_0 - \left(\frac{Aq\mu}{C_0 kh}\right) \div \left(\frac{Br_b^2 f\mu c}{kt_0}\right)$

‡ Note that none of the equations which apply to pressure build-up will be strictly applicable while the effect of the after-production is still significant.

APPENDIX B — LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS		
		Part I	Part II	
			Metric System	U.S. System
A	A numerical constant	—	9.517	70.60
a	Distance from well to fault	cm	m	ft
B	A numerical constant	—	717.6	948.2
C_o	Shrinkage factor, tank volume divided by subsurface volume	—	—	—
c	Compressibility	vols/vol/atm	vols/vol/kg/cm²	vols/vol/psi
D	A numerical constant	—	2870	3793
Ei	The exponential integral function (cf. footnote to page 2)	—	—	—
e	The exponential constant	—	—	—
F	A numerical constant	—	43.83	325.1
f	Porosity	a fraction	a fraction	a fraction
G	A numerical constant	—	21.91	162.6
h	Pay thickness	cm	m	ft
J_0	Bessel function of the first kind of order zero	—	—	—
J_1	Bessel function of the first kind of order unity	—	—	—
k	Permeability	darcy	md	md
ln	Logarithm to base e	—	—	—
$\log_{10}$	Logarithm to base 10	—	—	—
μ	Viscosity	cp	cp	cp
p	Pressure at distance r and time t	atm	kg/cm²	psi
p_o	Initial reservoir pressure, or reservoir pressure at moment of completion of well	atm	kg/cm²	psi
p_e	Final closed-in static pressure in the well	atm	kg/cm²	psi
p_w	Pressure in the well during build-up	atm	kg/cm²	psi
p^*	False extrapolated pressure	atm	kg/cm²	psi
Q_b	Total inflow over circle $r = r_b$	cc of subsurface volume	—	—
q	Rate of production of well, assumed constant †	cc of subsurface volume per sec.	m³/day	bbl/day
q_o, q_1, q_2, q_3	Well production rates associated with t_1, t_2, t_3 (see fig. 1)	cc of subsurface volume per sec.	—	—
r	Distance from well centre-line	cm	m	ft
r_b	Radius of external (assumed circular) reservoir boundary	cm	m	ft
r_w	Radius of well	cm	—	—
ρ	Density of reservoir fluid at pressure p	gm/cc	—	—
ρ_o	Density of reservoir fluid at pressure p_o	gm/cc	—	—
t	Time (measured from time zero at moment of completion of well)	sec	hour	hour
t_c	Value of t_o corrected for non-constancy of q †	sec	hour	hour
t_o	Value of t at moment of closing in the well †	sec	hour	hour
t_1, t_2, t_3	Various times associated with q_o, q_1, q_2, q_3 (see fig. 1)	sec	—	—
θ	Time elapsed since closing in the well	sec	hour	hour
u	An independent variable	—	—	—
u_1	$\frac{r_b^2 f\mu c}{4kt_o}$	—	—	—
x	An independent variable	—	—	—
x_n	The n^{th} root (in order of increasing magnitude) of $J_1(x) = 0$	—	—	—
y	The y-function, defined by $y(u) = Ei(-u) + \frac{1}{u}e^{-u}$ (See fig. 10)	—	—	—

† For methods of correction when q has not been kept constant, see paragraph I, 3b.

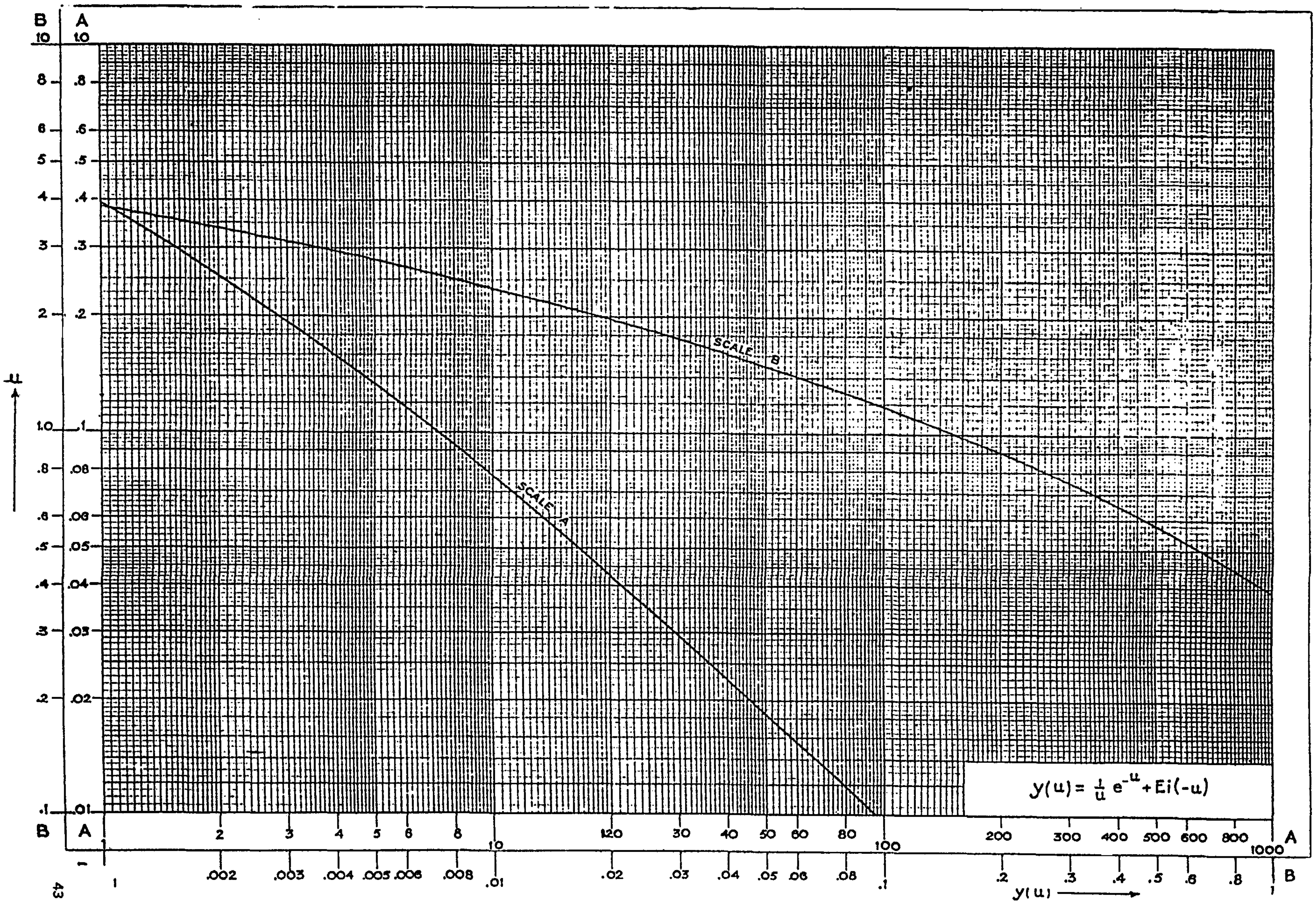


Fig. 10. Graph of the function y(u).