Analysis of Pressure Build-Up and Flow Test Data

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Abstract

Methods of using pressure build-up or flow tests to estimate formation permeability, formation pressure and well damage are reviewed. A number of phenomena which cause actual pressure build-up behavior to differ from the idealized case, such as the effects of boundaries, skins, inhomogeneities, partial penetration and two-phase flow, are discussed. It is concluded that the same method of analysis of build-up curves can be applied with slight modification to oil reservoirs, gas reservoirs and reservoirs producing both oil and gas. The calculations can be carried out on a form sheet, a copy of which is included in the paper. Examples are worked out for four different types of reservoirs.

Introduction

Although the basic theory of pressure build-up behavior in wells was developed many years ago, important contributions since that time have extended the original applicability to a much wider variety of situations. The purpose of the present paper is to summarize the present status of pressure build-up theory and of its applicability.

The approach in this paper will be to start with the simplest type of pressure build-up curve and to show how reservoir rock properties, reservoir fluid properties and well bore conditions tend to distort the idealized picture. Methods for taking these distortions into account and for determining values of reservoir formation properties from build-up curves will then be considered.

Build-up Curves

Infinite Homogeneous Reservoir

The theory for the pressure build-up behavior of a well producing a single, slightly compressible fluid from an infinite homogeneous reservoir was presented by Horner. According to him, the equation for build-up when oil is the only phase flowing is

\[ p_\infty = p_i - \frac{162.6 \, q \mu B}{k h} \log \frac{t + \Delta t}{\Delta t} \]  

where \( p_\infty \) = pressure in well during build-up, psi.

\( p_i \) = pressure in well after infinite closed-in time in infinite reservoir, psi (see Nomenclature for more general definition),
\( q \) = oil-production rate at surface conditions, B/D,
\( \mu \) = oil viscosity, cp,
\( B \) = oil formation volume factor,
\( k \) = permeability of formation to oil, md,
\( h \) = sand thickness, ft,
\( t \) = flowing time of well, and
\( \Delta t \) = closed-in time of well.

Derivation of the factor 162.6 is given in the Nomenclature. A graph of this equation is shown in Fig. 1, along with actual field data for a new well in an oil reservoir. For this case, the theory and practice agree well. To analyze this curve, note that the slope of the curve is equal to the coefficient of the logarithm term in Eq. 1. Therefore,

\[ k h = \frac{162.6 \, q \mu B}{\text{slope in psi/cycle}} \]  

Extrapolation of the straight-line section to an infinite shut-in time, \([ (t + \Delta t)/\Delta t = 1 \], gives the value for \( p_i \), as shown.

Bounded Homogeneous Reservoir

The pressure build-up behavior of a well in a bounded homogeneous reservoir is given by

\[ p_\infty = p_i - \frac{162.6 \, q \mu B}{k h} \left[ \log \frac{t + \Delta t}{\Delta t} \right] \]

Fig. 1—Pressure build-up for a nearly ideal reservoir (from Horner).

SPE 1631-G
The equation differs from Eq. 1 only by the addition of the term \( Y(t) \), which may be called the boundary effect and is a function of the shape of the drainage boundary of a well and of the production time. For a square drainage boundary, the boundary effect causes the pressure build-up curve to bend over as shown by the field example in Fig. 2. The extrapolated value from the straight-line section is called \( p_i \); in a bounded reservoir, \( p_i \) is best defined as the extrapolated pressure (see Nomenclature). The final static value of the pressure is called \( p_f \) the average pressure. A plot of \( (p - p_f)/(70.6 \mu B/\text{kh}) \) is given in Fig. 3 for one well in the center of a square drainage area. It may be seen that the difference between \( p_f \) and \( p \) increases as production time increases.

The function \( Y(t) \) is related to the function plotted in Fig. 3 by

\[
Y(t) = \frac{0.00331 \mu t}{\phi \mu c A} - \frac{p_i - p_f}{70.6 \mu B/\text{kh}}.
\]

The quantity \( (p - p_f)/(70.6 \mu B/\text{kh}) \) is plotted in Ref. 3 for a number of cases. In that paper it is called \( (P^* - p_f)/(q\mu A/k\text{h}) \). If there are other wells in a reservoir, the effect of production at other wells is to cause a well to be surrounded by a drainage boundary. On one side of this boundary fluid flows toward that well, and on the other side toward another well. For some time after a well is closed in, it can be treated as if its drainage boundary still exists. Thus, a well surrounded by other wells will have a build-up curve qualitatively similar to that of one well in a bounded reservoir. For very long closed-in times this is not true (see “Interference Tests”). Values for the average pressure \( p_f \) for reservoirs which have not been closed in long enough for the “bend over” shown in Fig. 2 to be observed may be obtained by the method of Ref. 3.

Wellbore Damage

As just mentioned, the effect of production at other wells is to flatten the build-up curve at long times. Quite logically then, the effect of wellbore damage would be to distort the curve at early closed-in time, as shown in Fig. 4 (from Ref. 4). The effect of wellbore damage on pressure distribution is to cause an extra drop right at the wellbore, as illustrated in Fig. 5 (from Hurst). This extra drop in pressure has been called a skin effect. Shortly after shut-in, the well pressure should rise by the amount \( \Delta p_{\text{skin}} \) shown on Fig. 5. Thus, the order of magnitude of the skin effect should be evident from the difference between flowing pressure and the pressure shortly after shutting in. Quantitatively, the skin effect \( S \) as defined by van Everdingen can be calculated from

\[
S = 1.151 \left[ \frac{P_{1 \text{ hour}} - p_i}{\Delta p} \right] \log \left( \frac{k}{\phi \mu c A} \right) + 1.07,
\]

and the pressure drop in the skin or damaged zone near the well by

\[
\Delta p_{\text{skin}} = \Delta p \times 0.87S.
\]

The value for \( P_{1 \text{ hour}} \) must be taken on the straight-line

Fig. 3—Correction to \( p_i \) for well in square drainage area.

Fig. 4—Pressure build-up showing effect of wellbore damage and after-production.

Fig. 5—Pressure distribution in a reservoir with a skin (from Hurst).
portion of the pressure build-up curve. If the build-up curve is not straight at one hour, it is necessary to extrapolate the straight-line portion backwards, as shown in Fig. 4. Note that a scale for \((t + \Delta t)/\Delta t\) and also a scale for \(\Delta t\) are used to facilitate extrapolation to \(p_i\), the instant. A measure of the efficiency of completion can be obtained by comparing the actual productivity index \(J\) and the ideal (no skin). The ratio of these two quantities is

\[
\text{Flow Efficiency} = \frac{J_{\text{actual}}}{J_{\text{ideal}}} = p_i - p_f - \Delta p_{\text{skin}}.
\]  

This is quite similar to the condition ratio of Gladfelter, et al. Although the use of \(p_i\) in this equation is strictly correct only for an infinite reservoir, the error introduced is small.

**Well Fill-up Effect**

The idealized theory, as exemplified by the straight line on Fig. 1, assumes that a well is closed in at the sand face itself and that, after closing in, no production at all enters the wellbore. In practice, however, a well is closed in at the surface, and for an instant after this time fluid continues to flow into the wellbore at an unchanged rate. Only after this fluid accumulates in the bore is the effect of closing-in at the surface transmitted to the formation. For this reason there is a lag in the build-up at early times, as shown by Fig. 6 (adapted from Ref. 7). If both gas and oil are flowing into the wellbore, the rise of gas to the top of the well during shut-in and the fall of oil to the bottom can cause the pressure to "hump", as shown in Fig. 7 for a well from South Texas. One of the ways of decreasing these wellbore effects is to use a recently developed tool which closes in the well at the bottom. This tool allows interpretation of build-up curves after a much shorter closed-in time.

**Effect of Reservoir Heterogeneity**

A reservoir consisting of stratified layers of different permeability was shown theoretically to have a "tail" to the build-up curve. An example of this for a well which is completed in two strata of different permeability is shown in Fig. 8. The pressure build-up takes place in both layers at early times, giving a straight-line section as shown. The slope of this straight-line section gives the total \(kh\) for both layers. After build-up is complete in the most permeable layer, the less permeable layer - which is at a higher average pressure - begins to feed fluid into the more permeable layer. This causes the rise above the straight-line section. Finally, an equalization will occur, and the curve will flatten as indicated by the dotted line.

Fissured limestone reservoirs also show this type of build-up. In fact, it appears that most reservoirs with two distinct types of permeability can show such behavior. The tail is greatest when the permeability difference is greatest, as shown by the example for a fissured limestone in Fig. 9.

**Gas Wells**

Aronofsky and Jenkins have shown that gas wells behave in a manner very similar to oil wells. With acceptable accuracy, the build-up equation is given by

\[
p = p_i - 162.6 \frac{q_{y/g} B_g}{k_i h_i} \log \left( \frac{t + \Delta t}{\Delta t} \right)
\]

where

\[
B_g = Z \frac{T}{T_i} \left[ \frac{p_{b/g}}{(p_i + p_f)/2} \right] = \text{gas formation volume factor at arithmetic average reservoir pressure}.
\]

Note that \(B_g\) is computed at the arithmetic average reservoir pressure. This average changes during build-up, so the slope of the curve of \(p\) vs \(\log [(t + \Delta t)/\Delta t]\) should also change slightly. This change will usually be negligible, and it will usually be satisfactory to approximate \(p\) by \(p_i\) in the equation for \(B_g\). When the gas equation is used in the form of Eq. 7, its analogy with the oil equation (Eq. 1) is emphasized. In Eq. 7, the gas flow rate needs to be expressed in barrels per day at standard conditions. Trube has given curves for gas compressibility \(c_g\). An example of a gas-pressure build-up curve is shown in Fig. 10. The form of the gas equation used by many authors may be obtained by substituting for \(B_g\) in Eq. 7.
Two-Phase Flow Effect

The shape of the pressure build-up curves when both oil and gas are flowing is very much the same as when only one phase flows. The interpretation is also much the same. Eq. 2 is also applicable during two-phase flow, as was justified by Perrine. The skin effect and wellbore damage can be calculated from Eqs. 4, 5 and 6, provided that the total mobility \( k_o / \mu_o + k_g / \mu_g + k_w / \mu_w \) is used instead of \( k_o / \mu_o \). The interpretation is also much the same as when oil and water are flowing is very much the same as when oil, gas and water are used instead of that for oil. Similar methods can be used for obtaining average pressure.

At high drawdown, the flow of gas can introduce an apparent skin effect even when there is none. An example of a pressure and a saturation distribution curve for a reservoir in which both oil and gas are flowing is shown in Fig. 11. A flow efficiency of about 70 per cent is calculated for this well, when actually there is no skin at all. The higher gas concentration around the well bore causes the apparent damage, as was shown by Perrine.

Partially Penetrating Wells

Nisle and Brons and Marting have studied the pressure behavior of such wells. Their work shows that the effect of partial penetration is to introduce an apparent wellbore damage. The amount of extra pressure drop caused by partial penetration can be found from Ref. 16.

Fall-off Curves

In water-injection wells it is natural to determine formation properties from a fall-off curve determined by closing in the well. The methods already discussed in this paper for analysis of build-up curves can also be used for fall-off curves if the mobilities \( k / \mu \) of water and oil are about the same. However, when the data for a fall-off curve are plotted in the usual manner, it is often difficult to determine just where the straight-line section begins and ends, as in the example in Fig. 2. A better method for analysis of these curves is to use the log \( (p - \bar{p}) \) vs. \( \Delta t \) plot, as shown in Fig. 12. This same plot may be used both when oil and water differ widely in properties and when they do not.

The method of plotting these curves is similar to that for layered reservoirs. It depends on finding, by trial and error, a value of \( \bar{p} \) which makes the fall-off curve linear at large time. This value is shown to be 230 psi in Fig. 12. This type of plot works at large times because the well pressure is approaching the average pressure exponentially. Values of \( k_h \) and skin factor can be obtained from the slope \( \beta \), and the intercept \( b \). Ref. 17 should be consulted for details of the method.

Interference and Flow Tests

Interference Tests

When one well is shut in and its pressure is measured while others in the reservoir are produced, the test is termed an interference test. This type of test can give information on reservoir properties that cannot be obtained from pressure build-up tests. In particular, values of the porosity can be obtained from these tests, whereas they cannot be obtained from pressure build-up. The simplest method of interpretation is based on superposition of the effects of each of the producing wells at the well in question. When the boundary of the reservoir is near, it also needs to be taken into account (for example, by imaging the producing wells). Collins has described a method for analysis of interference tests. Another method is given as follows.

When the outer boundary effect can be neglected, the pressure at the observation well is given by

\[
p = p_i - \frac{162.6 \mu_i B}{k_h} \log \frac{\Delta t + \Delta t}{\Delta t} + \frac{70.6 \mu_i B}{k_h} \left[ q_i \left( \frac{\phi \mu_c a_i}{0.00105 k_i} \right) + q_j \left( \frac{\phi \mu_c a_j}{0.00105 k_j} \right) + \ldots \right],
\]

Interference Tests

Fig. 9—Build-up in a fissured limestone reservoir (from Pollard).}

Fig. 10—Build-up in a gas well.

Fig. 11—Pressure and saturation distribution when both oil and gas are flowing.

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where \( q_1 \) is the production rate of Well No. 1, \( q_2 \) the production rate of Well No. 2, etc.; \( t_1 \) and \( t_2 \) are production times of these wells; \( a_1 \) and \( a_2 \) are distances from Wells 1 and 2 to the observation well. Variations in rates \( q_1 \) and \( q_2 \) may be taken into account by the principle of superposition.\(^1\) Eq. 8 is strictly applicable only to single-phase flow and, thus, is best used when reservoirs are above the bubble point. The log term in this equation gives the effect of producing and shutting-in the observation well itself. The \( Ei \) terms give the pressure drop at the distance \( a \) or \( a_i \) from the production Wells 1 or 2. Elkins\(^2\) has presented a good example of use of interference tests. In tight rocks it may be necessary to shut-in a well for a long time to observe the effects of interference. An example is shown in Fig. 13. The dotted line in this figure was obtained by extrapolating the linear portion on the log \([\theta (t + \Delta t)/\Delta t]\) plot, as shown on Fig. 14. To determine the magnitude of \( \phi \mu c/k \) by this proposed method, it is necessary that the difference between the extrapolated curve and the observed points be measurable with reasonable accuracy — the magnitude of this difference is equal to the interference terms (the \( Ei \) terms) in Eq. 8. If the difference is zero, then the contribution of the \( Ei \) terms is zero and \( \phi \mu c/k \) cannot be determined. A difference of 40 to 50 psi is necessary in order for \( \phi \mu c/k \) to be determined with some reliability.

The quantity \( \phi \mu c/k \) can be obtained by assuming values for this quantity, substituting in Eq. 8 and computing the observed pressures. The value of \( \phi \mu c/k \) which gives the best fit to the observed pressures is assumed to be applicable to the reservoir. From known values of \( \mu \) and \( c \), a value for \( \phi/k \) can be obtained. Since \( kh \) can be obtained from the slope of the pressure build-up curve, a value for \( \phi h \) can be obtained as the product of these two quantities.

Interference tests require considerably more time than pressure build-up tests, especially in low-permeability reservoirs. Note that a minimum of two months was required for the case shown in Figs. 13 and 14. This reservoir had a permeability of a few millidarcies.

**Flow Tests**

Flow tests can be interpreted in much the same manner as pressure build-up tests. In particular, the slope of the curve of flowing pressure vs flowing time is the same as the slope of the curve of shut-in pressure vs shut-in time. Wellbore damage can also be determined in a similar manner. A method for determining reservoir size from a flow test has been described by Jones.\(^3\) The method depends on determining the point at which the plot of pressure vs log (flowing time) departs from linearity. The method is useful for estimating minimum size of oil accumulations without sacrifice of production, as is necessary if a well is closed in for determination of average pressure.

**Discussion**

**Time Required for Build-up and Interference Tests**

A well should be closed in for pressure build-up long enough to allow the straight-line section indicated on Fig. 6 to be clearly delineated. The longest time will be required for deep, low-productivity, pumping wells not equipped with packers, because in these a long period of "after-production" will be needed to fill the wellbore with liquid and compressed gas. Only after the influx into the wellbore
wellbore becomes small can the simple theory of pressure build-up be applied. A simple method of determining when the rate of influx has become small is to determine casing-head pressure (CHP) and tubing head pressure (THP), as well as bottom-hole pressure (BHP) during build-up. The difference between BHP and THP or CHP is directly proportional to the mass of fluid between these two levels. A curve showing (BHP-THP) and (BHP-CHP) can be plotted. When the slope of this difference curve falls to, say, only 10 per cent of its initial value, the influx is probably small enough that the simple theory can be applied. At this time, the influx rate into the wellbore is only 10 per cent of the production rate at time of closing-in.

Miller, Dyes and Hutchinson have shown that boundary and interference effects begin to distort the straight-line section of a pressure build-up curve at a time \( \Delta t_a = 0.000264k \Delta t/\Phi \mu Cr^2 \), of the order of 0.1. This gives an upper limit to required shut-in times. Most of the points which will be used in pressure build-up analysis should fall in the range of dimensionless time \( \Delta t_p \) from 0.005 to 0.1. Interference effects begin to distort the straight-line portion of the production rate at time of closing-in.

Selection of Wells for Production Stimulation

As discussed by Gladfelter, et al., there are three main causes of low-productivity wells — (1) a “skin” near the wellbore, (2) low permeability throughout the reservoir and (3) lack of pressure. By taking a pressure build-up on a well prior to a well-stimulation treatment, it is possible to determine which of these three are causing the low productivity. If the trouble is a skin, then a wash treatment or a polyalcohol polymer will be required. If the difficulty is due to lack of pressure, some type of injection such as of water or gas will be required to expel the oil. Because they aid in diagnosis of the difficulty, pressure build-ups should be used as shown.

Example Calculations

Oil Reservoir

A pressure build-up curve for an oil reservoir above the bubble point is given in Fig. 4. After the slope of the curve is measured and other pertinent data are entered on the example calculation of Table 1, the calculations for \( k, S \) and flow efficiency can be carried out as shown.
Two-Type Permeability Reservoir

The type of pressure build-up curve shown in Fig. 15 cannot be analyzed completely by the foregoing method. In particular, the curve cannot be extrapolated to give \( p \), as was the build-up on Fig. 4. This extrapolation should be made by using the type of plot shown on Fig. 16. A value for the average pressure \( \bar{p} \) is estimated, and log \((p - \bar{p})\) is plotted vs \( \Delta t \) the closed-in time. The value of \( p \) which gives the best straight line is the value for the average pressure in the drainage radius of the well. Note that, for values of \( \bar{p} \) which are too small, the plot of log \((p - \bar{p})\) vs \( \Delta t \) bends down. For values which are too large, the curve bends upward. Sometimes this upward bend is difficult to detect, and all the plots appear to be straight when \( p \) is above a certain value. In such case it is usually satisfactory to take \( \bar{p} \) as the lowest value for which the curves become straight. A value of \( kh \) for both layers is obtained from the slope of the earlier straight-line section indicated in Fig. 15. An approximation for the skin effect is obtained from the same equations as for a homogeneous reservoir.

An example calculation for a two-type permeability reservoir is shown in Table 2. For this case \( p \), and \( \bar{p} \) are approximately equal, but this is often not true. It should be emphasized that the skin effect and flow efficiency obtained by the method shown are only approximately correct for the two-type permeability reservoir. The maximum dimensionless shut-in time is

\[
\Delta t_p = 0.000264 \cdot (23.7) \cdot 49 = 0.066.
\]

This is within the interval 0.005 to 0.1, so the correct portion of the build-up was used.

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**Fig. 16—Pressure build-up in a heterogeneous reservoir.**

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**Gas Reservoir**

The form sheet used for oil reservoirs can also be used for gas reservoirs. It is only necessary to convert the gas rate in cubic feet per day to barrels per day by dividing by 5.615. The gas formation volume factor is obtained from

\[
B_g = Z_T \frac{T_o}{T_r} \frac{p_g}{p_r^2} \frac{0.1272}{0.000264} = 0.0056.
\]

The method of obtaining \( B_g \), \( c_g \) and \( p_g \) is shown in Table 3A. The rest of the analysis for Fig. 10 is straightforward. For this example,

\[
\Delta t_p = 0.000264 \cdot (6.9) \cdot 24 = 0.0056.
\]

Although many of the build-up points are outside the recommended interval, a study of Fig. 10 leaves little doubt that the correct section was analyzed. In cases such as this it is often valuable to plot (BHP-THP) vs time to ensure that the influx rate is small at the end of build-up.

**Reservoir Producing Below Bubble Point**

Calculations for these reservoirs can be handled on the same form sheet as that for the oil reservoir. The only

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**TABLE 2—EXAMPLE OF SUBSURFACE PRESSURE CALCULATIONS FOR TWO-TYPE PERMEABILITY RESERVOIR.**

<table>
<thead>
<tr>
<th>Test Data:</th>
<th>Hole Size (in.)</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cum. Prod. ( N_o ) (bbl)</td>
<td>2390</td>
<td></td>
</tr>
<tr>
<td>Stabilized Daily Prod. ( q ) (bbl)</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>Effective Life ( t ) (hours)</td>
<td>( 24N_0/q ) 441</td>
<td></td>
</tr>
</tbody>
</table>

1. Calculation of \( kh \) (md-ft) and \( k \) (md)

\[
h = 27 \text{ ft} \quad a = 0.37 \\
\frac{q}{a} = 130 \text{ bbl/day} \\
\frac{\Delta p \text{ (cycle)}}{17} = 3.99 \\
\frac{h}{a} = 162.6 \times (130) \cdot (0.37) \cdot (1.39) = 639.5 \text{ md-ft} \cdot k = (639.5) \cdot (27) = 23.68 \text{ md}.
\]

2. Calculation of Skin Effect \( S \) and Pressure Drop Due to Skin \( \Delta p \) (psi)

\[
k = 23.68 \text{ md} \\
\frac{\rho}{a} = 130 \text{ bbl/day} \\
\frac{\Delta p \text{ (cycle)}}{17} = 2620 \text{ psi} \\
\frac{\rho}{a} = 1.5 \times 10^{-2} \text{ psi/psi} \\
\Delta p \text{ skin} = (17) \times 0.87 = 14.65 \text{ psi}.
\]

3. Calculation of Productivity Index \( J \) (B/D/psi) and Flow Efficiency

\[
J_{\text{act}} = \frac{q}{(p_i - p_f)} \\
J_{\text{id}} = \frac{q}{(p_i - p_f - \Delta p \text{ skin})} \\
\Delta p \text{ skin} = 1008 \text{ psi} \\
\frac{q}{a} = 130 \text{ bbl/day} \\
\frac{\Delta p \text{ skin}}{2620 \text{ psi}} = 0.1125 \text{ B/D/psi} \\
J_{\text{act}} = \frac{(3775) - (2620)}{(130)} = 0.1125 \text{ B/D/psi} \\
J_{\text{id}} = \frac{(1155) - (1008)}{0.8844} = 0.1272
\]

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**TABLE 3A—EXAMPLE SHOWING CALCULATION OF \( B_g, \mu_g \) and \( c_g \) FOR A GAS WELL.**

\[
Z_T = \frac{T_o}{T_r} \frac{p_g}{p_r^2} \frac{0.809}{14.65 \text{ psi}} = 0.00561.
\]

Using \( T_o \) and \( p_g \) and a gas gravity of 0.8, \( \mu_g \) is obtained from Ref. 21. Figs. 4 and 6, as \( \mu_g = 1.7 \times 0.00115 = 0.0024 \text{ cp.} \) Also, \( c_g \) is obtained from Truempel as 0.73/663 = 0.000347 \( \text{ psi}^{-1} \).
change necessary is to calculate the total mobility and total compressibility. Calculations for the curve shown in Fig. 17 are made in Table 4A. The total mobility and the total compressibility are then used on the form sheet in calculating $S$. These steps are exemplified in the example calculation of Table 4. In this case, 

$$\Delta t = \frac{0.000264 (92.9)}{24 (0.15) 8.5 \times 10^4 (500)} = 0.018.$$ 

Since this time is in the interval 0.005 to 0.1, the correct portion of the data was used in build-up analysis.

### Conclusions

It appears that a single form sheet can be adopted with slight modification to the analysis of nearly any pressure build-up curve. Pressure build-up behavior now appears to be reasonably well understood.

### Nomenclature

- $A$ = drainage area of well, sq ft
- $a_i$, $a_o$ = distance between observation well and production well (No. 1 or No. 2), ft
- $B$ = oil formation volume factor
- $B_o$ = gas formation volume factor
- $c$ = compressibility, psi$^{-1}$
- $h$ = formation thickness, ft
- $J$ = productivity index, B/D/psi
- $k$ = formation permeability, md
- $p_c$ = pressure obtained when linear portion of pressure build-up curve, psi
- $p(t + \Delta t)$ = closed-in time, dimensionless
- $\phi$ = porosity, fraction
- $E_i = 1 - x = \frac{c^2}{s}$

### Table 3—Subsurface Pressure Calculations for a Gas Well

<table>
<thead>
<tr>
<th>Test Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size [in.]: 7</td>
</tr>
<tr>
<td>Cum. Prod. $N_c$ (bbl): $1.19 \times 10^7$ (6390 MMcF)</td>
</tr>
<tr>
<td>Effective Prod. Life $t$ (hour) = $24N_c/q$ (50.6 × $10^4$)</td>
</tr>
</tbody>
</table>

| Stabilized Daily Prod. $q$ (bbl): 536,900 (0.07 MMcF/D) |

1. Calculation of $kh$ (md-ft) and $k$ (md)

<table>
<thead>
<tr>
<th>$h$</th>
<th>$f$</th>
<th>$q$</th>
<th>$B_o$</th>
<th>$k$</th>
<th>$\phi$</th>
<th>$\Delta t$ (1 cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$536,900$</td>
<td>$8/0$</td>
<td>$\mu_o$</td>
<td>$0.000811$</td>
<td>$12159$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$kh = 162.6 \times (536,900) \times (0.000811) = \Delta t$ (1 cycle) = 579.5 md-ft;

$k = \frac{(579.5)}{(84)} = 6.90 \text{ md}$.

The remainder of the analysis is straightforward.

### Table 4A—Example Showing Calculation of Total Mobility ($k/p_c$) and Total Compressibility $c_r$ for a Reservoir Below Bubble Point

<table>
<thead>
<tr>
<th>Test Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size (in.): 12</td>
</tr>
<tr>
<td>Effective Prod. $q$ (bbl): 924 a.l., 15.38 MMcF gas (7.740 MM bbl gas)</td>
</tr>
</tbody>
</table>

| Stabilized Daily Prod. $q$ (bbl): 298 cu ft/bbl or 53.1 lbm/bbl |

1. Calculation of $kh$ (md-ft) and $k$ (md)

<table>
<thead>
<tr>
<th>$h$</th>
<th>$f$</th>
<th>$q$</th>
<th>$B_o$</th>
<th>$k$</th>
<th>$\phi$</th>
<th>$\Delta t$ (1 cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$298$</td>
<td>$8/0$</td>
<td>$\mu_o$</td>
<td>$0.675$</td>
<td>$229$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$kh = 162.6 \times (298) \times (0.675) = \Delta t$ (1 cycle) = 922 md-ft;

$k = \frac{922}{229} = 4.09 \text{ md}$.

### Table 4—Subsurface Pressure Calculations for Reservoir Below Bubble Point

<table>
<thead>
<tr>
<th>Test Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size (in.): 12</td>
</tr>
<tr>
<td>Effective Prod. $q$ (bbl): 924 a.l., 15.38 MMcF gas (7.740 MM bbl gas)</td>
</tr>
</tbody>
</table>

| Stabilized Daily Prod. $q$ (bbl): 298 cu ft/bbl or 53.1 lbm/bbl |

1. Calculation of $kh$ (md-ft) and $k$ (md)

<table>
<thead>
<tr>
<th>$h$</th>
<th>$f$</th>
<th>$q$</th>
<th>$B_o$</th>
<th>$k$</th>
<th>$\phi$</th>
<th>$\Delta t$ (1 cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$298$</td>
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<td>$0.675$</td>
<td>$229$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$kh = 162.6 \times (298) \times (0.675) = \Delta t$ (1 cycle) = 922 md-ft;

$k = \frac{922}{229} = 4.09 \text{ md}$.

### Table 4B—Example Showing Calculation of Total Mobility ($k/p_c$) and Total Compressibility $c_r$ for a Reservoir Below Bubble Point

<table>
<thead>
<tr>
<th>Test Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size (in.): 12</td>
</tr>
<tr>
<td>Effective Prod. $q$ (bbl): 924 a.l., 15.38 MMcF gas (7.740 MM bbl gas)</td>
</tr>
</tbody>
</table>

| Stabilized Daily Prod. $q$ (bbl): 298 cu ft/bbl or 53.1 lbm/bbl |

1. Calculation of $kh$ (md-ft) and $k$ (md)

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<th>$q$</th>
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<th>$k$</th>
<th>$\phi$</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>$8/0$</td>
<td>$\mu_o$</td>
<td>$0.675$</td>
<td>$229$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$kh = 162.6 \times (298) \times (0.675) = \Delta t$ (1 cycle) = 922 md-ft;

$k = \frac{922}{229} = 4.09 \text{ md}$.

### Table 4C—Example Showing Calculation of Total Mobility ($k/p_c$) and Total Compressibility $c_r$ for a Reservoir Below Bubble Point

<table>
<thead>
<tr>
<th>Test Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size (in.): 12</td>
</tr>
<tr>
<td>Effective Prod. $q$ (bbl): 924 a.l., 15.38 MMcF gas (7.740 MM bbl gas)</td>
</tr>
</tbody>
</table>

| Stabilized Daily Prod. $q$ (bbl): 298 cu ft/bbl or 53.1 lbm/bbl |

1. Calculation of $kh$ (md-ft) and $k$ (md)

<table>
<thead>
<tr>
<th>$h$</th>
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<th>$\Delta t$ (1 cycle)</th>
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<td>$229$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$kh = 162.6 \times (298) \times (0.675) = \Delta t$ (1 cycle) = 922 md-ft;

$k = \frac{922}{229} = 4.09 \text{ md}$.

### Fig. 17—Pressure build-up in a reservoir when both oil and gas are flowing.
Subscripts

\[
\begin{align*}
 a &= \text{atmospheric (60°F, 14.65 psia)} \\
 f &= \text{flowing} \\
 i &= \text{infinite time} \\
 o, w, g &= \text{oil, water, gas}
\end{align*}
\]

Dimensionless Quantities

Dimensionless time in the Darcy System of Units (darcy, sec, cp, cm, atm) is \(t_D = \frac{kt}{\phi \mu c r_e^2}\). In this system of units in this report (md, hr, cp, ft, psi), this quantity is \(t_D = \frac{0.000264}{\phi \mu c r_e^2}\). The constant 0.00105 in Eq. 8 is 4 (0.000264). In the Darcy System of Units, the flow rate is usually written \(qp_B/4\pi kh\), when it is desired to express it in units of atmospheres. In units in this report, this quantity is 70.6 \(qp_B/kh\). Also,

\[70.6 \frac{qp_B}{kh} \log \frac{t}{kh} = 162.6 \frac{qp_B}{kh} \log \frac{D}{D_0}.
\]

References


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